

Steady State and Dynamic Optimization of the Drive Systems with Surface-Mounted Permanent Magnet Synchronous Machines

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Abstract—The paper deals with a problem not approached in the literature: the achievement of a control system that ensures optimization in both steady state and dynamic operation. The problem refers to a drive system with a surface mounted permanent magnet synchronous machine. The aim is to minimize the power dissipated on the stator winding in steady state, respectively the energy dissipated in dynamic mode. The main difference that appears refers to the current limitation. A solution for an adequate control system is indicated. The results are validated by simulations.

Keywords—steady state optimization, optimal control, drive system, SMPMSM

I. INTRODUCTION

The optimization of the electrical drive systems usually refers to the minimization of energy losses both in steady state and in dynamic operation. The methods used in the two cases are somewhat similar, but also different in some aspects.

The optimization refers, as a rule, to the minimization of Joule losses, which are predominant in all cases. The only losses that can still be reduced by controlling the machine are those in iron ones, but these are not taken into account because they are smaller. An additional justification for the steady state is that, in the case of analytical procedures (frequently used), the errors due to this neglect are of the same order of magnitude as those due to the not knowing the exact parameters of the machine. Moreover, the neglect is justified in dynamic operation, when the currents have high values.

The present paper refers to the optimization of the drive systems with surface-mounted permanent magnets synchronous machines (SMPMSM). There are a large number of papers that deal with the optimization of the steady state for such electric machines, among which we mention [1], [2], [3], [4], [5], [6], [7]. Comprehensive syntheses for this problem can be found in [8], [9], [10]. Regarding the optimal control of dynamic operation of the indicated type systems, we can mention [11], [12], [13], [14], [15].

The paper approaches a situation when it is necessary to introduce a control that ensures both steady-state and dynamic operation, a problem rarely addressed in the literature (e.g. [16]), but for the same limitation of currents in both operating modes). As will be seen, there are differences in the formulation of the constraints and the control system will have to distinguish between the two cases.

In what follows, aspects related to the optimization of the steady state and the dynamic operation, a comparison between the two cases, simulations results and final conclusions are presented.

II. STEADY STATE OPTIMIZATION

A. General conditions

Stator voltages equations in a d - q rotor reference frame are

$$\begin{aligned} u_d(t) &= R i_d(t) + L \frac{di_d(t)}{dt} - p \omega(t) L i_q(t) \\ u_q(t) &= R i_q(t) + L \frac{di_q(t)}{dt} + p \omega(t) L i_d(t) + p \omega(t) \Phi \end{aligned} \quad (1)$$

where R and L are the stator resistance and inductance, ω is the rotor speed, p is the pole pairs number. The above equations contain the d - q components of the instantaneous stator voltages and currents vectors $\mathbf{u} = [u_d \ u_q]^T$, $\mathbf{i} = [i_d \ i_q]^T$ (T denotes the transposition).

The electromagnetic torque developed by a SMPMSM is

$$m(t) = \frac{3}{2} p \Phi i_q(t) \quad (2)$$

where Φ is the flux created by the permanent magnet in the synchronous reference frame.

The equations (1) become in steady state:

$$\begin{aligned} u_d(t) &= R i_d(t) - p \omega(t) L i_q(t) \\ u_q(t) &= R i_q(t) + p \omega(t) L i_d(t) + p \omega(t) \Phi \end{aligned} \quad (3)$$

Two dual optimization problems can be formulated: minimization of electrical power losses and maximization of torque per current. The second problem has no significance if the speed is below the rated one, because in this situation only the i_q component is different from zero and then the torque/current ratio is constant, according to (2). For speeds higher than the rated one, a flux-weakening operation is required, which can be achieved by limiting the current and/or voltage. If the electric machine operates at maximum current and at the torque imposed by the load, the problem of maximizing the torque/current ratio is meaningless. The problem makes sense if only the voltage limitation intervenes. Moreover, for similar reasons, as it will be seen, similar aspects intervene in the case of the problem of minimum power consumption.

In the following, we will only deal with the problem of minimizing the dissipated power, which consist in

minimizing the function

$$f(\mathbf{i}) = \|\mathbf{i}\|^2 = i_d^2 + i_q^2 \quad (4)$$

under the conditions of compliance with the equality restriction

$$h(\mathbf{i}) = \frac{3}{2} p \Phi i_q - m = 0 \quad (5)$$

which comes from (2) and with the inequality constraints related to current and voltage:

$$g_1(\mathbf{i}) = i_d^2 + i_q^2 - i_M^2 \leq 0 \quad (6)$$

$$g_2(\mathbf{u}) = u_d^2 + u_q^2 - u_M^2 \leq 0 \quad (7)$$

where i_M and u_M are the maximal admissible values for current and voltage, respectively.

Some additional special constrains may appear in certain applications, but they will not be addressed here. The last restriction can be formulated according to the currents i_d and i_q replacing (3) in (7). The simple calculations lead to

$$g_2(\mathbf{i}) = (i_d + c_1(\omega))^2 + (i_q + c_2(\omega))^2 - r^2(\omega) \leq 0 \quad (8)$$

where

$$\begin{aligned} c_1(\omega) &= p^2 \cdot \omega^2 L \Phi / Z^2, \quad c_2(\omega) = p \cdot \omega R \Phi / Z^2 \\ r(\omega) &= u_M / Z, \quad Z^2 = R^2 + (p\omega L)^2 \end{aligned} \quad (9)$$

Inequalities (6) and (8) represent surfaces bounded by circles in the $i_d - i_q$ plane, Fig.1.

The circle defined by (6) has its centre in origin and constant radius. Inequality (8) corresponds to circles with centres in $(-c_1(\omega), -c_2(\omega))$ and radius $r(\omega)$. For very small ω , the centre of the circle approaches the origin and the radius tends towards the u_M/R value (significantly higher than the rated current value). The radius of the circles decreases with the increase of ω , and the position of the centre changes according to ω on a curve located in the third quadrant. If the resistance R is neglected, the centre of the circle is in $(-\Phi/L, 0)$.

The steady state optimization problem is formulated as follows: determine i_d and i_q which minimize the function $f(i_d, i_q)$ given by (4), respecting constrains (5), (6) and (8). To solve the problem, one can use the Karush-Kuhn-Tucker method [17], as it is done in [3]. For this purpose, the synthetic Lagrange function is formulated:

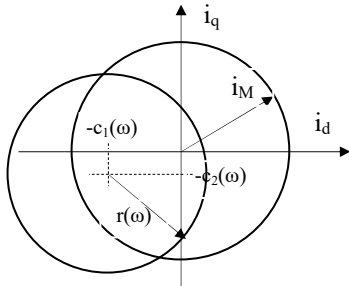


Fig. 1. $i_d - i_q$ restrictions

$$L(\mathbf{i}) = f(\mathbf{i}) + \lambda h(\mathbf{i}) + \sum_{j \in I} \mu_j g_j(\mathbf{i}) \quad (10)$$

The necessary minimum condition is the cancellation of the gradient ($\partial L / \partial \mathbf{i} = 0$) of the function (10) with reference only to the active constraints:

$$\frac{\partial f(\mathbf{i})}{\partial \mathbf{i}} + \lambda \frac{\partial h(\mathbf{i})}{\partial \mathbf{i}} + \sum_{j \in I} \mu_j \frac{\partial g_j(\mathbf{i})}{\partial \mathbf{i}} = 0, \quad (11)$$

where I is the set of active restrictions, for which we have $\mu_j > 0$. In the present case, there is one of the following four possibilities: $I = \emptyset$, $I = \{1\}$, $I = \{2\}$, $I = \{1, 2\}$.

Taking into account the forms of the functions f , h , g_1 and g_2 , the condition (11) becomes

$$\begin{aligned} (1 + \mu_1 + \mu_2) i_d + \mu_2 c_1(\omega) &= 0 \\ (1 + \mu_1 + \mu_2) i_q + \mu_2 c_2(\omega) + \frac{3}{2} \lambda \Phi &= 0 \end{aligned} \quad (12)$$

To these equations is added the equality restriction (5) and, as the case may be, the equations:

$$\begin{aligned} g_1(\mathbf{i}) &= 0 \\ g_2(\mathbf{i}, \omega) &= 0 \end{aligned} \quad (13)$$

Next, we will discuss the four cases indicated above for active constraints.

B. *No current and voltage constraints* ($I = \emptyset$, $\mu_1 = 0$, $\mu_2 = 0$)

It follows from equations (5) and (12):

$$\begin{aligned} i_d &= 0, \quad i_q = \frac{2}{3} \frac{m}{p \Phi} \\ \lambda &= -\frac{8}{9} \frac{m}{p^2 \Phi^2} \end{aligned} \quad (14)$$

There is only component i_q that depends on the load torque and can vary within the limits $[0, i_M]$. Usually i_M is the rated current i_N and corresponds to the electric machine's rated load. In this situation, the speed that is established in steady state is the rated speed and is reached if the rated voltage is applied (usually chosen as the maximum voltage u_M). It follows that the circle (8) that passes through the point $(0, i_M)$ corresponds to the rated speed and voltage. The limit speed obtained for $i_q = i_M$ is called the base speed (ω_b). It is often equal to the rated one or close to it. Above this speed, the flux-weakening operation mode is entered.

C. *Current active constraint* ($\mu_1 > 0$, $\mu_2 = 0$)

In this case, from (5) and (12) the currents i_d and i_q result in the form (14), specifying that $i_q = i_M$. It also follows from (12)

$$i_M (1 + \mu_1) = -\frac{3}{4} \lambda p \Phi \quad (15)$$

which indicates a linear dependence between the multipliers λ and μ_1 , respectively an equivalence of the two restrictions (the maximum torque corresponds to the maximum current).

D. Voltage active constraint ($\mu_1=0, \mu_2>0$)

In this case, it follows from (5) and (8)

$$i_d = [r^2(\omega) - (\frac{2}{3} \frac{m}{p\Phi} + c_2(\omega))^2]^{1/2} - c_1(\omega) \quad (16)$$

$$i_q = \frac{2}{3} \frac{m}{p\Phi}$$

and using equations (12), the expressions of the multipliers λ and μ_2 can be established.

E. Current and voltage active constraint ($\mu_1>0, \mu_2>0$)

The operating point corresponds to the intersection of the circles $g_1(\mathbf{i})$ and $g_2(\mathbf{i}, \omega)$ given by (6) and (8), resulting

$$i_d = (1/c_1(\omega))(k^2 - 2c_2(\omega)i_q) \quad (17)$$

$$i_q = \frac{2}{3} \frac{m}{p\Phi}$$

where

$$k^2(\omega) = r^2(\omega) - c_1^2(\omega) - c_2^2(\omega) - i_M^2 = \frac{u_M^2 - \omega^2 p^2 \Phi^2}{Z^2} - i_M^2 \quad (18)$$

Using the equations of the restrictions (all of the equality type in this case), the expressions of the multipliers can be established.

F. Remarks

- The operation in the first two cases corresponds to what is called maximum torque per ampere (MTPA). The torque/ampere ratio is constant in this case, according to (2).
- In the cases (D) and (E), the flux-weakening operation takes place, as an effect of component i_d . This occurs in the cases where it is desired to reach a speed higher than ω_b .
- The component i_q has the value imposed by the load torque in steady state in all cases. The operation point is established on the circle (8), for imposed ω and i_q . As such, the required value for the component i_d results from (6). The same speed can be achieved for the same torque and for a lower voltage. This operation can be achieved by increasing the i_d component (as absolute value), that is, a decrease of the flux compensates the voltage drop in order to maintain the speed. But the increase of i_d means that the minimum for current, respectively the minimum of losses, is no longer obtained.
- At high speeds, the limit values i_M and u_M are reached (see point (E)). The operation takes place in a point on the circle (6) for i_q imposed by the load. It is possible that a circle (8) corresponding to a speed lower than the desired one passes through this point. Obviously, increasing the speed can only be done in the case of sufficiently small load torques.
- The speed can be increased as much as possible for very small load torques only if the machine parameters ensure the fulfillment of condition

$\Phi/L > i_M$. Otherwise, the component i_d can be increased (in absolute value) only up to $i_d = \Phi/L$. In this way, a different flux-weakening operation is obtained, in which the i_d component is limited.

III. DYNAMIC OPTIMIZATION

A. Unconstrained optimal control

The optimal control problem for the considered drive system takes into account equations (1), where u_d and u_q are control variables, and i_d , i_q and ω are state variables. A significant simplification can be obtained by considering the currents as control variables and speed as state variable. This approach is justified by the fact that it is possible to obtain a very fast variation of the currents, which thus satisfy the requirements imposed on control variables. Besides, in many cases, the control of the drive systems is realized by currents.

The performance index for the problem of minimization of Joule losses is

$$I_d = \frac{1}{2} \int_{t_0}^{t_f} (i_d^2(t) + i_q^2(t)) dt \quad (19)$$

The mechanical equilibrium equation is

$$d\omega(t)/dt = [m_e(i_q, t) - m(\omega, t)] / J \quad (20)$$

where J is the inertia momentum and m is the load torque. It results from (20):

$$\omega_f = \omega(t_f) = \omega(t_0) + (t_f - t_0)(m_{em}(i_q) - m_m) / J \quad (21)$$

where

$$m_{em} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} m_e(i_q(t)) dt, \quad m_m = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} m(t) dt \quad (22)$$

are the mean values of the electromagnetic and load torque, respectively. We will consider $t_0 = 0$, a free final moment t_f and a fixed final speed ω_f .

Relationship (21) shows that the problem can be solved only if the variation of the load torque is known on the optimization interval. The solution can be established if at least the form of the variation of the load torque is known and the magnitude of the load torque is estimated at the beginning of the interval. This condition will be considered fulfilled in the following. Because it interests the mean value of the load torque, it will be considered a certain variation on the optimization interval, but independent of speed. For the SMPMSM, the electromagnetic torque is given by (2). The operation will be considered only below the rated loading and speed.

To establish the condition of optimality, the Hamiltonian [18] of the problem is formed

$$H = \frac{1}{2}(i_d^2 + i_q^2) + \frac{\lambda}{J}(m_e - m) \quad (23)$$

where λ is the co-state variable.

The minimum necessary conditions for (20) are:

$$\begin{aligned} \frac{\partial H}{\partial i_d} = 0 &\Rightarrow i_d = 0 \\ \frac{\partial H}{\partial i_q} = 0 &\Rightarrow i_q = -\frac{\lambda c}{J}, c = \frac{3}{2} \Phi \\ \frac{\partial H}{\partial \omega} = -\frac{d\lambda}{dt} &\Rightarrow \frac{d\lambda}{dt} = 0 \Rightarrow \lambda = \text{const.} \end{aligned} \quad (24)$$

It results from the last two relationships $i_q = \text{constant}$ and (21) becomes

$$\frac{J}{t_f}(\omega_f - \omega_0) = -\frac{c^2 \lambda}{J} \quad (25)$$

and one obtains

$$\lambda = -\frac{J}{c^2} \left[\frac{J}{t_f}(\omega_f - \omega_0) - m_m J \right], \quad \omega_0 = \omega(t_0) \quad (26)$$

The control variable results from (22.2):

$$i_q = \frac{1}{c} \left[\frac{J}{t_f}(\omega_f - \omega_0) - m_m J \right] = \frac{1}{c} (J \varepsilon - m_m) \quad (27)$$

where ε is the angular acceleration (constant on the optimization interval).

Since $i_d = 0$ and i_q is constant, the performance index (19) can be expressed as:

$$I_d = \frac{t_f}{2} i_q^2 \quad (28)$$

The transfer time results from (20):

$$t_f = \frac{J(\omega_f - \omega_0)}{m_{em} - m_m} \quad (29)$$

Thus, the performance index is

$$I_d = \frac{J}{2} \frac{\omega_f - \omega_0}{c i_q - m_m} i_q^2 \quad (30)$$

The minimum necessary condition $\partial I_d / \partial i_q = 0$ leads to

$$c i_q^* = 2 m_m \quad (31)$$

Therefore,

$$m_e^* = 2 m_m \quad (32)$$

that is, the optimal electromagnetic torque must be the double of the mean load torque. In other words, the dynamic accelerating torque must be equal with the load one:

$$J \frac{\omega_f - \omega_0}{t_f} = m_m, \quad \text{or} \quad J \varepsilon = m_m \quad (33)$$

In this case, the transfer time is

$$t_f = \frac{J(\omega_f - \omega_0)}{m_m} \quad (34)$$

Calculating the second order derivative for (30), it is found to be positive, so that the sufficient minimum condition is satisfied.

Similar results were obtained by authors for other electric machine types [19].

B. Problems with constraints

- *Current restriction* can be imposed when it is possible to achieve dangerous current values. If during the acceleration period the average load torque has at most the value of the rated torque, then, according to (31), the current will have at most the double value of the rated current i_N . Exceeding of the mentioned value of the mean load torque indicates a wrong choice of electric machine power and corrections must be made in this regard. Therefore, one introduce the restriction:

$$i_q \leq 2 i_N \quad (35)$$

The electric machine supports this kind of current increase during the short time of the transient period and, anyway, the energy dissipated during the acceleration period is minimal for the adopted optimal control. Considering those mentioned, the theoretical optimization calculations must not take into account the current limitation, because, normally, it must be fulfilled.

Problems can arise in connection with the choice of the inverter, because it must ensure a current greater than i_N . Instead, the electric machine power can be chosen lower and the investment expenses are balanced, remaining the advantage of lower operating expenses.

Anyway, the limiting of the current to its rated value can cause malfunctions. For example, the electric machine cannot start if the load torque is equal to m_N . In general, if the load torque is close enough to m_N , the acceleration takes a long time, due to too little dynamic accelerating torque. The solution in such cases is to adopt a higher power electric machine, or to accept, at least for the dynamic operation, a current greater than i_N .

- *Time restrictions:* It is necessary to introduce in some applications an inferior limit for the duration of the transient operation. But more frequently situations may appear in which this duration is superior limited. In fact, from (34), large values for t_f result if the load torque is small (the reduced value of the dynamic accelerating torque determines a very low acceleration). As such, the value of t_f must be limited, but it is more useful and convenient to impose an inferior limit for acceleration ε . As a quasi-optimal solution, the procedure described in point (A) can be used, requiring that ε be greater than a convenient value.

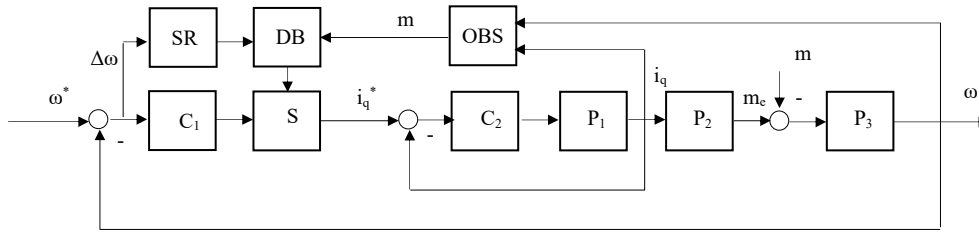


Fig. 2. Structure of the optimal system

IV. IMPLEMENTATION OF THE OPTIMAL SYSTEM AND SIMULATION RESULTS

The general structure of a speed control of an electrical drive system that ensures the optimization of both the steady state and the dynamic operation is presented in Fig. 2.

Comparative with a usual speed control system, some additional blocks must be introduced in such optimal system:

- a saturation amplifier with variable saturation level (S);
- an element that notices the operation in stationary or dynamic mode (SR) of the system; it can be based on the measurement of the speed error $\Delta\omega = \omega^*(t) - \omega(t)$ (ω^* is the prescribed speed and $\omega(t)$ is the measured speed); if $\Delta\omega$ is 0, the operation is in steady state mode and the decision block (DB) sets the current limitation to the i_N value; if $\Delta\omega$ is not zero, the operation is in transient mode and the current is limited to the $2 m_m/c$ value;
- an observer for the torque (OBS), e.g. [20].

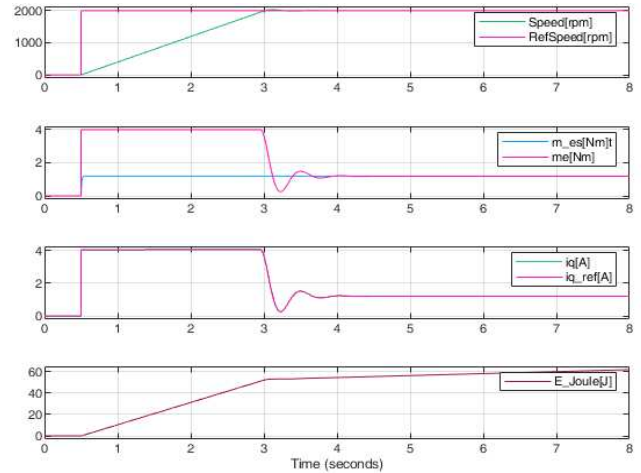
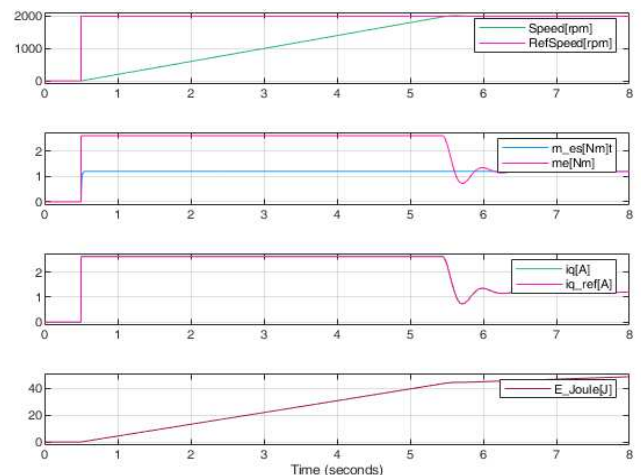
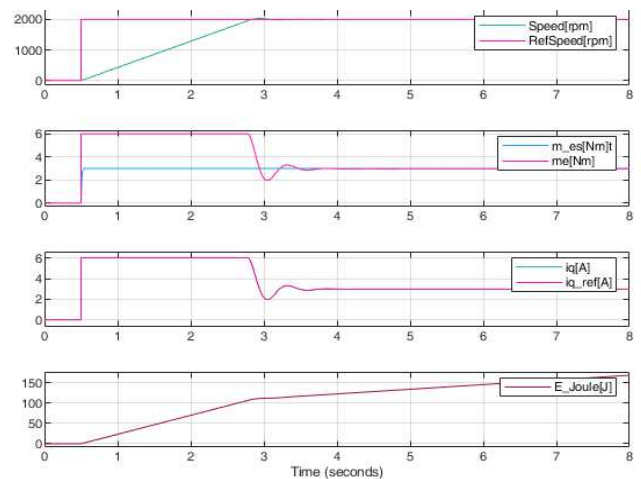
The mentioned elements are attached to a cascade control system, according to Fig.2, where C1 and C2 are PI type controllers, and P1, P2 and P3 are fixed parts of the system: inverter, electric machine, driven working machine. According to the mentioned functions, the decision block DB establishes the level of the signal from the output of the amplifier with saturation S. In the last part of the transient process, when $\Delta\omega$ has low values, the block S comes out of saturation, operating with an adequate proportionality factor and the whole assembly works as a usual cascade control system.

In order to validate the presented theoretical considerations some numerical simulations were performed. The following parameters (of the 8MSA4M SMPMSM type) were considered: $P=1930W$, $\omega_N = 4500rpm$, $m=4Nm$, $i_N=4.4A$, $R=1.275\Omega$, $L=7.25mH$, $\Phi=0.22Wb$, $J=0.034 kgm^2$, $p=3$.

Some results from the performed tests are presented in Fig.3, Fig.4 and Fig.5. In all figures are indicated: reference and motor speed, estimated load torque and electromagnetic one, component i_q of the stator current and dissipated energy, respectively.

Fig.3 and Fig.4 are for a load torque $m=0.3m_N$. In the first case, $m_c=m_N$ and in the second case, $m_c=2 \cdot m$ (optimal control). The losses energy in the second case is 83% from the losses energy in the non optimal control.

Fig.5 refers to an optimal control for $m=0.75 m_N$. The dissipated energy increases by 2.5 times comparatively with the case presented in Fig.4 (similar with increase of the load torque).


 Fig. 3. Transient operation: $i_M=i_N$, $m=0.3m_N$, $E_J=52.6J$

 Fig. 4. Transient operation: $i_M=0.6i_N$, $m=0.3m_N$, $E_J=43.7J$

 Fig. 5. Transient operation: $i_M=1.5 i_N$, $m=0.75m_N$, $E_J=111.8J$

V. CONCLUSIONS

- The optimization of a drive system with surface mounted permanent magnet synchronous machine is presented. The optimal system ensures the minimization of the dissipated power / energy.

- The solutions for steady state and dynamic optimization are indicated.

- The structure that ensures both steady state and dynamic optimization is indicated.

- Simulation results are presented.

- Further development of the presented approach refers to the dynamic optimization for the flux-weakening operation.

REFERENCES

- [1] M. Preindl, S. Bolognani, "Optimal state reference computation with constrained MPTA criterion for PM motor Drives," *IEEE Trans. on Power Electronics*, vol. 30, no. 8, pp.4524-4535, Aug. 2015.
- [2] L. Sepulchre, M. Fadel, M. Pietrzak-David, G. Porte, "MTPV flux-weakening strategy for PMSM high speed drive," *IEEE Trans. Ind. Appl.*, 2018, 54, 6081-6089.
- [3] H. Houmsi, F. Bribiesca-Argomedo, P. Massioni, R. Delpoux, "A Karush-Kuhn-Tucker approach to field-weakening for surface-mounted permanent magnets synchronous motors," *Intern. Conf. on Control, Automation and Diagnosis (ICCAD'23)*, May 2023, Rome.
- [4] Y. D. Yoon, S-K Sul, "New flux weakening control for surface mounted permanent magnet synchronous machine using gradient descent method," *7th Intern. Conf. on Power Electronics*, Oct. 2007, Daegu, Korea.
- [5] R. Nalepa, T. Orłowska-Kowalska, "Optimum trajectory control of the current vector of a nonsalient-pole PMSM in the field-weakening region," *IEEE trans. Ind. Electron.*, 2012, 59, 2867-2876.
- [6] A. Dianov, A. Anuchin, "Adaptive maximum torque per ampere control of sensorless permanent magnet motor drives," *Energies*, vol. 13, no. 19, Sep. 2020.
- [7] B. Cheng, T. R. Tesch, "Torque feedforward control technique for permanent-magnet synchronous motors," *IEEE Trans. Ind. Electron.*, vol. 57, no. 3, pp. 969-974, Mar. 2010.
- [8] N. Bianchi, P. G. Carlet, L. Cinti, L. Ortombia, "A review about flux-weakening operating limits and control techniques for synchronous motor drives," *Energies* 2022, 15.
- [9] A. Dianov, F. Tinazzi, S. Calligaro, S. Bolognani, "Review and classification of MTPA control algorithms for synchronous motors," *IEEE Trans. on Power Electronics*, vol.37, no. 4, April 2022.
- [10] Z. Li, D. O'Donnell, W. Li, P. Song, A. Balamurali, N. C. Kar, "A comprehensive review of state-of-the-art maximum torque per ampere strategies for permanent magnet synchronous motors," in *Proc. 10th Int. Elect. Drives Prod. Conf.*, 2020, pp. 1-8.
- [11] K-T Chang, T-S Low, T-H. Lee, "Optimal speed controller for permanent-magnet synchronous motor drives," *IEEE Trans. on Ind. Electronics*, 41 (5), pp. 503-510, Oct.1994.
- [12] A.G.Khiabani, A. Heydari, "Optimal torque control for permanent synchronous motors using adaptive dynamic programming", *IET Power Electronics*, July 2020.
- [13] Q. Wei, X Wang, X.P. Hu, "Optimal control for permanent magnet synchronous motor," *Journal of Vibration and Control*, vol. 20, Issue 8, 2014.
- [14] M. Gaiceanu, "Advanced control of the permanent magnet synchronous motor", in Ed. A. El-Shahat, *Electric machines for smart grids applications – Design, simulation and control*, IntechOpen, 2018
- [15] R. Molavi, D. A. Khaburi, "Optimal control strategies for speed control of permanent-magnet synchronous motor drives," *World Academy of Science, Engineering and Technology*, 44, 2008.
- [16] V. Smidl, S. Janous, L. Adam, Z. Peroutka, "Direct speed control of a PMSM drive using SDRE and convex constrained optimization," *IEEE Trans. on Ind. Electron.*, vol. 65, no.1, Jan. 2018, 532 – 542.
- [17] M. Aoki, *Introduction to optimization techniques*, University of California, Los Angeles, The Millan Company, 1974.
- [18] M. Athans, P.L. Falb, *Optimal Control*, McGraw Hill, New York, 1966.
- [19] C. Boțan , V. Horga, F. Ostafi, M. Albu, M. Rățoi, "General aspects of the electrical drive systems optimal control," *12th European Conf. on Power Electronics and Applications (EPE 2007)*, Aalborg, Denmark, Sept. 2007.
- [20] Y.A.Zorgani, Y.Koubaa, M.Boussak, "MRAS state estimator for speed sensorless ISFOC induction motor drives with Luenberger load torque estimation", *ISA Transactions*, 61, (2016)