Harmonics of the Component Formed by Pairs of Alternative Unitary Step, Symmetrical Regarding the Central Step of the PWM Inverter Output Voltage

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***Abstract*—The analytical approach of the output voltage of the PWM type inverter, from the composition of solar power plants, was possible by highlighting in the voltage oscillogram two types of waves: a step type voltage, alternative, centered on the middle of the alternation (AUS) and one of type pairs alternate unit steps, alternating, symmetrical to the central step (PAUS). The definition of these waves in relation to the functional quantities of pulse amplitude, conduction angles and priming delay angles led to the identification of their Fourier developments and then, on this basis, allowed the analytical determination of the optimal operating regimes of PWM inverters, and the possibility of establishing a convenient control principle. The paper deals with the harmonics of PUAS type components, which overlap with the central component, in order to reconstruct the real voltage wave, obtained by oscillation.**

Keywords—harmonics; Discrete Fourier Transform; PWM inverter; photovoltaic power plants

I. Introduction

As the photovoltaic elements, from the composition of the photovoltaic power plants (PPP) produce dc electricity, the use of inverters was resorted to, in order to supply to supply the power grid with ac. The oscillogram in Fig. 1 shows the shape of the output voltage $u_i(t)$ of a PWM inverter, within a PPP, where the amplitude of the dc voltage was noted with U_M and with T_c − the period of the voltage wave (its nominal value is 20 ms, for f = 50 Hz).

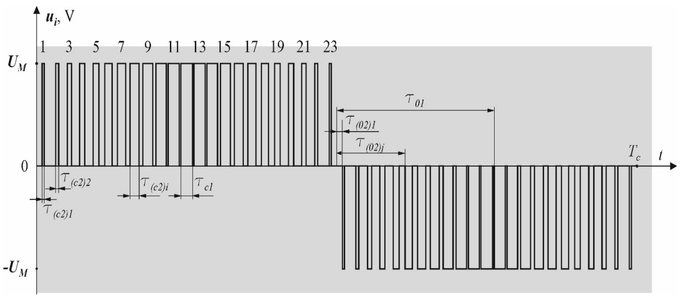

Fig. 1. Oscillogram of the line output voltage of a PWM type inverter.

The harmonic analysis of this wave [1] highlighted as more important harmonics those with the orders $N\in\{45;47;49\}$, as well as the fact that the decrease of the harmonics amplitudes with increasing their order is not strictly monotonous, there are several local minimum and maximum values. Also, the following values were determined for the main characteristic quantities of the harmonics [1−4], of the analyzed voltage wave:

- The rms of the voltage wave is U_e=0,768·U_M, V.
- The level of the fundamental, relative to the actual wave value, γ_{1e}=81,64%.
- The complete deforming residue at the ac wave, denoted U_{dca} [3], is equal, in this case, to the deforming residue of the higher harmonics U_{def} (because the continuous component U_0 is zero), both having the value $U_{def} = U_{dca}$ = 0.4435·U_M, V.
- THD, defined as the ratio, expressed as a percentage, between the complete deforming residue and the actual wave value, δ_{ae} = 57.75%.

As shown in [1,5], the PWM inverter output voltage oscillogram (Fig. 1, considered in absolute or relative values) can be decompose into two types of signals, as shown in Fig. 2:

- A step signal wave, of determined duration, centered in the middle of the respective alternation, called alternate unitary step (AUS) signal (Fig. 2, *a*).
- A unit amplitude wave, consisting of two pulses on each alternation, placed symmetrically with respect to the central pulse, referred to as a wave consisting of pairs of unitary, alternating steps (PAUS, Fig. 2, *b*).

Overlapping the two types of elementary waves, described above, results in a periodic, alternating wave, as shown in Fig. 2, *c*, in which the number of pulses on an alternation is three, of which one, central, of greater width and other two, of smaller widths, placed symmetrically with respect to the central pulse. This is a simplified form of the PWM inverter output voltage, in relative units (r.u.), because the amplitude of the signals is

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unitary (U_M = 1); by increasing the number of PAUS type signals, one can reach the real, oscillographed form (in r.u.). Thus, for the real voltage wave (Fig. 1), an AUS type wave would overlap with another 11 PAUS type waves.

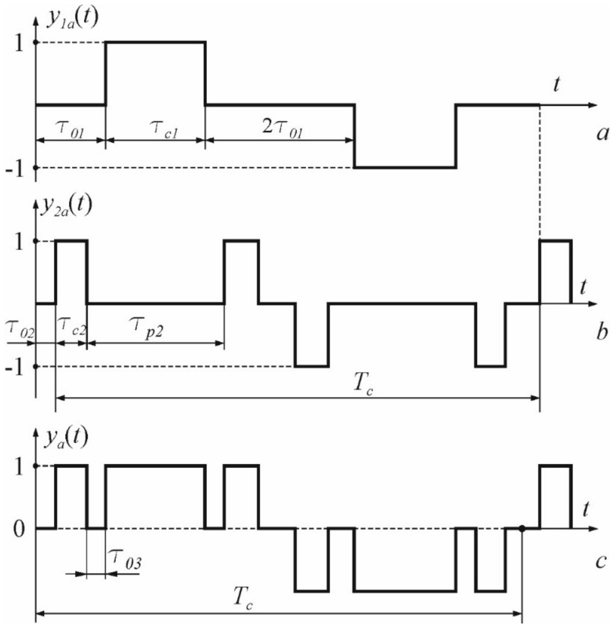

Fig. 2. Components of the output voltage of the PWM inverter: *a* − AUS wave; *b* − PAUS wave; *c* − the output voltage of the PWM inverter, in simplified form.

II. Analysis of PAUS Wave

A. Fourier decomposition of PTUA signal

The shape considered for the PAUS type signal (Fig. 2, *b*) is described analytically by the characteristic durations, such as: duration τ_{c2} of the unit impulse, duration of the initial pause τ_{02} and the period of the signal T_c, as well as by the unit amplitude, the latter corresponding to the notion of relative amplitude of the real wave. The function corresponding to the PAUS type signal, denoted $y_{2a}(t)$, being analytically defined, in portions [3], the Continuous Fourier Transform is applied, determining for the PAUS type signal the following Fourier development (in r.u.):

$$y_{2a}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k}{2k-1}\sin\frac{(2k-1)\pi\tau_{c2}}{T_c}\cdot \cos\frac{(2k-1)\pi(4\tau_{c2}+2\tau_{c2}-T_c)}{2T_c}\sin[(2k-1)\varpi_c t]. \quad (1)$$

The rms value of the fundamental, denoted simplified Y_{1e}, is determined from the (1), for k = 1, which, through some elementary transformations takes the form:

$$Y_{1e} = \frac{2\sqrt{2}}{\pi}\sin\frac{2\pi\tau_{c2}}{T_c}\sin\frac{2y(2\tau_{02}+\tau_{c2})}{T_c}, r.u. \quad (2)$$

By using (2), it is obtained that the maximum of the rms value of the fundamental results for $\tau_{c2} = T_c / 4$ and $\tau_{02} = 0$, at the value:

$$Y_{1eM} = \frac{2\sqrt{2}}{\pi} \cong 0.90032, r.u. \quad (3)$$

Given the fact that the rms value of a wave, as a global quantity, has an important role in the analysis of harmonics, because it facilitates the calculation or verification of some characteristic quantities of this regime, it is considered appropriate to determine this quantity a priori. The effective value of the $y_{2a}(t)$ wave, for which the notation $Y_{2a(e)}$ is proposed here, can be obtained using the analytical expression of the wave, defined in portions (as above, Fig. 2, *b*), using the integral calculation [3], based on the relation:

$$Y_{2a(e)} = \sqrt{\frac{1}{T_c}\int_0^{T_c} y_{2a}^2(t)\cdot dt} = 2\cdot\sqrt{\frac{\tau_{c2}}{T_c}}, r.u. \quad (4)$$

which reveals a value identical to that of the AUS signal [1] if there is a relation $\tau_{c1} = 2\tau_{c2}$. Relation (5) reveals a dependence of the rms value of the PAUS signal only on the conduction duration $2\tau_{c2}$, which is an obvious energetic justification.

B. Harmonics of PAUS type signal

The harmonics assessment of a voltage in the form of the PAUS signal is further carried out by studying the dependencies of the rms value of the fundamental $Y_{1e}(\tau_{c2},\tau_{02})$ and the deforming residue, of the higher harmonics, $Y_{def}(\tau_{c2},\tau_{02})$, in relation to the duration τ_{c2} of the unitary impulses and the duration of delay in priming τ_{02} (for simplicity, the "2*a*" index is omitted below, as only the PAUS type wave is treated) and the values are considered in r.u.; the real values will be obtained from the relative ones, by multiplying them by U_M.

For the impulse duration τ_{c2} the following range of distinct values is considered:

$\tau_{c2} \in$ {0,33; 0,66y; 1,0; 1,5; 2,0; 2,5; 3,0; 3,5; 4,0; 4,5; 5} ms,

and for the duration of delay in priming of the impulses – the set:

$\tau_{02} \in$ {0; 0,625; 1,25; 1,875; 2,5; 3,125} ms,

however, there is an upper limitation of the conduction duration τ_{c2} depending on the size of the delay in priming τ_{02}, as provided:

$$\tau_{c2} \le \frac{T_c}{4} - \tau_{02}, \text{ ms} \quad (5)$$

which leads to the maximum values $\tau_{c2M}(\tau_{02})$, of the conduction durations, according to the data presented in Table I.

TABLE I. MAXIMUM VALUES $\tau_{c2M}(\tau_{02})$ OF THE CONDUCTION DURATIONS, DEPENDING ON THE AMOUNT OF DELAY IN PRIMING τ_{02}, AT THE PAUS TYPE SIGNAL

τ_{02}, ms	0	0.625	1.250	1.875	2.500	3.125
τ_{c2M}, ms	5.000	4.375	3.750	3.125	2.500	1.875

Considering the maximum values of the conduction durations τ_{c2M}, at a given value of the delays in priming τ_{02}, we proceeded to determine the data set, presented in Table II, for the rms value of the fundamental $Y_{1e}(\tau_{c2}, \tau_{02})$, calculated with (2).

TABLE II. VALUES OF THE FUNCTION $Y_{1e}(\tau_{c2}, \tau_{02})$, IN R.U., DEPENDING ON THE CONDUCTION DURATION τ_{c2} AND THE DELAY ON PRIMING τ_{02}, FOR THE PAUS SIGNAL

τ_{02}, ms	τ_{c2}, ms										
	0.33	0.67	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4. 5	5.0
0.000	0.0096	0.0393	0.0860	0.1856	0.3111	0.4502	0.5893	0.7148	0.8143	**0.8783**	**0.9003**
0.625	0.0444	0.1067	0.1807	0.3108	0.4512	0.5882	0.7082	**0.7997**	**0.8536**	0.8647	-
1.250	0.0723	0.1579	0.2479	0.3887	**0.5227**	**0.6366**	**0.7194**	0.7629	-	-	-
1.875	0.0893	**0.185**	**0.2774**	**0.4075**	0.5146	0.5882	0.621	-	-	-	-
2.500	**0.0927**	0.184	0.2646	0.3642	0.4281	0.4502	-		-	-	-
3.125	0.0819	0.1549	0.2116	0.2655	-	-	-		-	-	-

A first, more important observation is that, at a given value of the conduction duration τ_{c2}, there is a maximum of the size Y_{1e} (the respective values are bolded in Table II), located relatively closer to the upper limit of the range of possible values of the priming delay duration τ_{02}, which can be exploited to achieve the optimum functional parameters of the PWM inverter. It also confirms the maximum possible value of the rms value of the fundamental, $Y_{1eM} = 0.9003$, identical to the value determined by canceling the derivatives of the function $Y_{1e}(\tau_{c2},\tau_{02})$, in relation to the variables τ_{c2} and τ_{02} (rel. (3)). The 3D graphical representation of the function, presented in Fig. 3, provides a complete picture of the dependence of the rms value of the fundamental Y_{1e}, on the two variables involved, τ_{c2} and τ_{02}.

In order to expressly report the characteristic sizes for the harmonics, the rms values Y_e (abbreviated notation for $Y_{2a(e)}$) of the PAUS wave are determined, according to the values established for the conduction duration τ_{c2} (rel. (4)), obtaining the data in Table III. There is a steady increase in the rms value of the Y_e wave as the conduction time τ_{c2} increases, reaching τ_{c2} = 5 ms, i.e. when the two impulses in an alternation cover the entire duration of the alternation, to obtain the maximum rms value of the wave, $Y_{eM} = 1$.

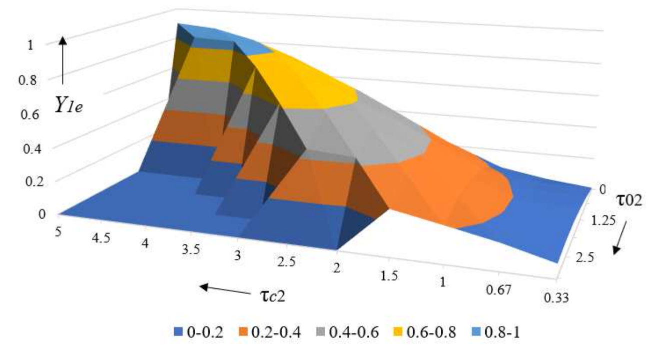

Fig. 3. 3D graphical representation of the dependence of the PAUS signal fundamental (rms value), on the command variables τ_{c2} and τ_{02}.

Having determined the Y_e rms values (rel. (4), Table III) and the rms values of the fundamental harmonic Y_{1e} (rel. (2) and Table II), the deforming residue of the higher harmonics Y_{def} is determined, as a global quantity, with the relation:

$$Y_{def} = \sqrt{Y_e^3 - Y_{1e}^2} \text{ ms} \quad (6)$$

so that, for the same values of the independent variables, τ_{c2} and τ_{02}, the data set presented in Table IV, on the basis of which the following more important observations are made:

- The Y_{def} deforming residue of the PAUS type wave has a specific variation in relation to the conduction duration τ_{c2}, for a constant value of the delay in priming, τ_{02} = const.; thus, for example, for τ_{02} = 0.625 ms (second line in Table III), the deforming residue increases constantly from the value Y_{def} = 0.253 (at τ_{c2} = 0.33 ms), to Y_{def} = 0.4432 (at τ_{c2} = 2.0 ms), then decreases to an absolute minimum Y_{def} (τ_{c2} = 3.5 ms) = 0.2459, followed by a new increase towards the end value of the domain, Y_{def} (τ_{c2} = 4.5 ms) = 0.3903.
- The absolute minimum of the deforming residue, $Y_{def(\min)} = 0.2396$, is recorded for τ_{02} = 2.5 ms and τ_{c2} = 0.33 ms (highlighted in red, non-thickened, in Table III).
- The absolute maximum of the deformation residue, $Y_{defM} = 0.5507$, is recorded for τ_{02} = 0 ms and τ_{c2} = 2 ms (highlighted in red, bold, in Table III).

A particular variation of the deforming residue is noticed, for τ_{c2} = 2.5 ms, when it registers the highest, equal values, Y_{def} = 0.5453, at the ends of the variation interval of the delay at priming, τ_{02} = 0 ms and respectively τ_{02} = 2.5 ms (black, bold, in Table III).

TABLE III. FUNCTION VALUES $Y_E(\tau_{c2})$ AS A FUNCTION OF CONDUCTION DURATION τ_{c2}, FOR THE PAUS SIGNAL

τ_{c2}, ms										
0.33	0.67	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4. 5	5.0
0.2569	0.3661	0.4472	0.5477	0.6325	0.7071	0.7746	0.8367	0.8944	0.9487	1

TABLE IV. THE DEFORMING RESIDUE VALUES $Y_{def}(\tau_{c2},\tau_{02})$ AS A FUNCTION OF CONDUCTION DURATION τ_{c2} AND OF DELAY IN PRIMING τ_{02}, FOR PAUS SIGNAL

τ_{02}, ms	τ_{c2}, ms										
	0.33	0.67	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4. 5	5.0
0.000	0.2567	0.3639	0.4389	0.5153	**0.5507**	**0.5453**	0.5028	0.4349	0.3699	0.3586	0.4352
0.625	0.2530	0.3502	0.4091	0.4510	0.4432	0.3925	0.3137	0.2459	0.2671	0.3903	--
1.250	0.2465	0.3303	0.3722	0.3859	0.3561	0.3078	0.2872	0.3434	-	-	-
1.875	0.2409	0.3159	0.3508	0.3660	0.3677	0.3925	0.4629	-	-	-	-
2.500	0.2396	0.3165	0.3605	0.4091	0.4655	**0.5453**	-	-	-	-	-
3.125	0.2435	0.3317	0.394	0.4791	-	-	-	-	-	-	-

C. *Reconstitution of the PAUS signal with different harmonic numbers*

As with the AUS signal, it was considered important to reconstruct the PAUS signal based on the expression determined for its Fourier development. The following numbers have been chosen for the summation index "k" in (1): $k_{Max} \in \{5,10,30,50\}$.

For these values of the summation index, the maximum numbers of the summed harmonics, for the reconstitution of the PAUS wave, are respectively

$$N_{\text{Max}} \in \{9; 19; 59; 99\} \quad (7)$$

The aspects of the PAUS wave, reconstituted for the maximum numbers of harmonics above, are represented in Fig. 4, revealing on the one hand the correctness of the analytical determinations and − on the other hand − the need of a number of at least 100 harmonics for the accurate reconstruction of PAUS wave.

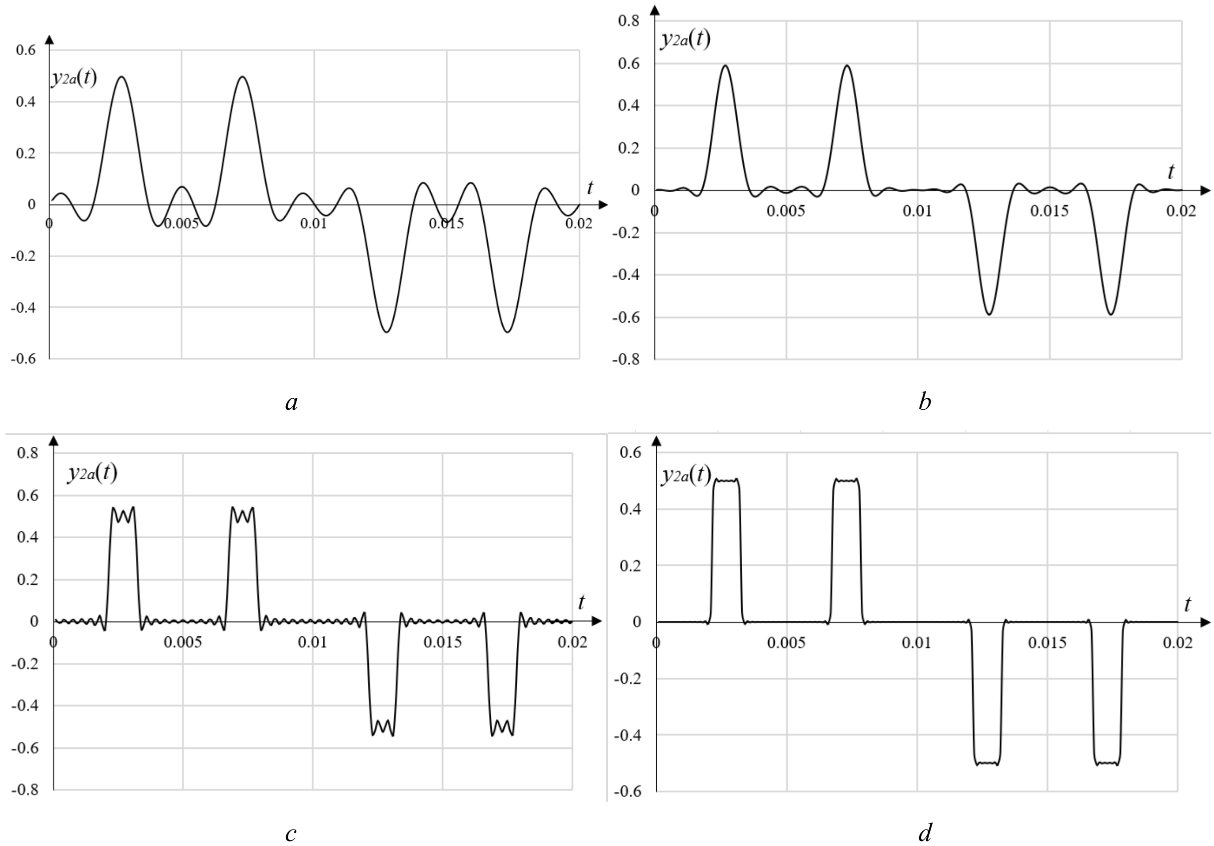

Fig. 4. Aspects of the PAUS wave, for different NMax numbers of harmonics, considered, for reconstitution: *a* − NM = 9, *b* − NM = 19, *c* − NM = 59, *d* − NM = 99.

III. CONCLUSIONS

The connecting element between the photovoltaic panels and the ac network is represented by the PWM inverter, whose output voltage has, during an alternation, the appearance of equal amplitudes impulses, but with different durations. The values of these durations have significant meanings both in terms of energy and power quality [6−8]. The analysis of the output voltage of the PWM inverter, as well as the identification of some control principles, applied to it, represent the main objectives of the research undertaken in the paper.

The main possibility of the output voltage decomposition of the PWM inverter into two types of signals, the first consisting of an alternating unitary step signal (referred to by the acronym AUS) and a second signal, consisting of pairs of alternating unitary steps (PAUS), proved to be constructive, the results of the analysis undertaken bringing analytical justifications for some phenomena observed by the Discrete Fourier Transform of the real output voltage, of the PWM inverter.

For the PAUS waveform, analyzed in more detail in this paper, it was found that the absolute maximum of the rms value of the fundamental, Y_{1e}, occurs in the case of "degeneration" of the PAUS signal into AUS signal, which occurs when the two pulses on the durations $\tau_{c2} = T_c/4 = 5$ ms each have an alternation, and the delay in priming is zero, $\tau_{02} = 0$.

The dependence of the harmonic coefficients on the control sizes, such as the conduction times and the delay times in priming, makes it possible to identify some control criteria considered advantageous for the different energy situations, which appear in the relation PPP − energy system [9−11].

One of the most important contributions of the analytical approach undertaken is the fact that the amplitudes of the harmonics, at the Fourier development of periodic, non-sinusoidal waves, do not decrease strictly monotonously with the increase of the harmonic orders, a fact that has been highlighted both theoretically and on an experimental basis.

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