Septic-convolution Kernel - Comparative Analysis of the Interpolation Error

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***Abstract*—The first part of the paper describes the septic-convolution (QC) interpolation kernel. The SC kernel consists of seventh-order polynomials and approximates the ideal sinc function in the interval [−4; 4]. The second part of the paper presents the results of the Experiment, which was realized with the aim of determining the interpolation error, MSE, when interpolating with the interpolation SC kernel. In addition, a comparative analysis with interpolation error, which occurred during interpolation with an interpolation fifth-order polynomial kernel (QC kernel), was performed. After that, the analysis of the values of the kernel parameters, α, determined theoretically and experimentally, is presented. Deviations of the theoretical values, in relation to experimental ones, were analyzed over statistical parameters (μ, σ^2). Finally, the efficiency of the algorithm for estimating the theoretical values of the kernel parameters, using parameters Accuracy and Precision, was determined. Experimental results are shown in tables and by graphs.**

***Keywords*—convolution, interpolation, interpolation kernel, septic-convolution kernel**

I. INTRODUCTION

For interpolation of band-limited signals, the ideal interpolation kernel is of the form *sin*(x)/x (in the notation *sinc*) where $-\infty \leq x \leq +\infty$ [1, 2]. The spectral characteristic of the *sinc* interpolation kernel is a rectangular function, H_{sinc}. The *sinc* kernel cannot be practically realized because it has infinite limits. For this reason, there is a need to truncate the *sinc* interpolation kernel to a finite length. As a consequence of the truncated *sinc* kernel, its spectral characteristic deviates from the ideal, rectangular, characteristic, which leads to: a) ripple in the passband and stopband, and b) finite slope in the transition band.

The idea is to approximate the truncated *sinc* interpolation kernel with a low-degree polynomial function. A polynomial zeroth-degree kernel (n = 0) allows interpolation by rounding to the nearest-neighbor [3, 4]. Nearest-neighbor interpolation is the most efficient in terms of computational speed, but, in doing so, the largest interpolation error is generated. A linear, first-degree interpolation kernel (n = 1) is described in reference [5]. A quadratic, second-degree interpolation kernel (n = 2) is described in references [3, 6]. A cubic, third-degree interpolation kernel (n = 3), intended for parametric cubic convolution, PCC, is described in references [1, 5, 7]. The parameterization of the cubic interpolation kernel, by introducing the kernel parameter α, is shown in references [1, 8, - 10]. By changing the value of the kernel parameter α, the characteristics of the kernel can be changed and, in this way, adjusted to the corresponding signal that is interpolated. The process of changing the kernel parameter for customization is called parameter optimization. Quintic-convolution (QC), fifth order interpolation kernel (n = 5), is described in reference [11]. Septic-convolution (SC), seventh order interpolation kernel (n = 7), is described in references [12, 13].

In this paper, the authors try to answer the questions: a) what is the value of the interpolation error when interpolating with the SC kernel, in relation to the interpolation error when interpolating with the QC kernel?, and b) is the theoretical optimal value of the kernel parameter, α_T, equal to the values of the optimal kernel parameters, α_E, which are determined by experimental interpolation of audio signals?. In order to answer the questions, the Experiment was realized. In the Experiment, interpolations of audio signals from the Test Database were performed. The Test base is created from the G1 - G7 tone signal. Tones G1 - G7 are played on Steinway B concert piano. The recording was performed in the acoustics laboratory of Iowa University. The interpolation results are presented in tables and graphs. In this paper, the interpolation error is defined via the Mean Square Error, MSE. Accuracy and Precision, as measures of deviation of the experimental optimal values of the kernel parameters, α_E, in relation to the theoretical

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optimal values of the kernel parameters, α_T, using statistical parameters, (μ, σ^2), were calculated.

The paper is organized as follows: Section II describes SC kernel. Section III describes the Algorithm for optimal kernel parameters estimating. Experimental results and comparative analysis are presented in Section IV. Section V is the Conclusion.

II. Septic-Convolution Kernel

The septic-convolution (SC) kernel consists of seventh-order polynomials and approximates the ideal *sinc* function in the interval [−4; 4]. The SC kernel is given by [11]:

$$r(x)=\begin{cases} a_7|x|^7+a_6|x|^6+a_5|x|^5+a_4|x|^4+ & |x|\le 1 \\ a_3|x|^3+a_2|x|^2+a_1|x|+a_0; & \\ b_7|x|^7+b_6|x|^6+b_5|x|^5+b_4|x|^4+ & 1<|x|\le 2 \\ b_3|x|^3+b_2|x|^2+b_1|x|+b_0; & \\ c_7|x|^7+c_6|x|^6+c_5|x|^5+c_4|x|^4+ & 2<|x|\le 3, \\ c_3|x|^3+c_2|x|^2+c_1|x|+c_0; & \\ d_7|x|^7+d_6|x|^6+d_5|x|^5+d_4|x|^4+ & 3<|x|\le 4 \\ d_3|x|^3+d_2|x|^2+d_1|x|+d_0; & \\ 0; & |x|>4 \end{cases} \tag{1}$$

where α is the kernel parameter. The SC kernel coefficients are:

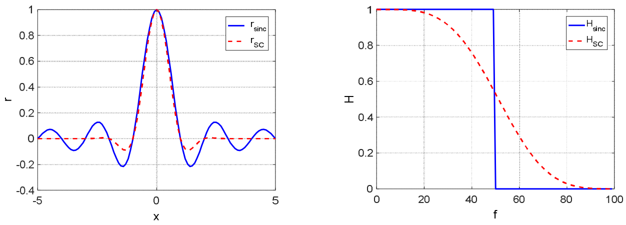
$$\begin{cases} a_0=1, & a_1=0, \\ a_2=-384\alpha-\dfrac{1393}{578}, & a_3=0, \\ a_4=760\alpha+\dfrac{1960}{867}, & a_5=0, \\ a_6=-621\alpha-\dfrac{1148}{867}, & a_7=245\alpha+\dfrac{821}{1734}. \end{cases}$$

$$\begin{cases} b_0=-2352\alpha-\dfrac{2233}{1156}, & b_1=14168\alpha-\dfrac{120407}{6936}, \\ b_2=-36000\alpha-\dfrac{13006}{289}, & b_3=47880\alpha+\dfrac{127575}{2312}, \\ b_4=-35640\alpha-\dfrac{128695}{3468}, & b_5=14952\alpha+\dfrac{32683}{2312}, \\ b_6=-3309\alpha-\dfrac{2492}{867}, & b_7=301\alpha+\dfrac{1687}{6936}. \end{cases}$$

$$\begin{cases} c_0=-47280\alpha-\dfrac{8505}{1156}, & c_1=133336\alpha+\dfrac{42525}{2312}, \\ c_2=-157632\alpha-\dfrac{5670}{289}, & c_3=101640\alpha+\dfrac{1575}{136}, \\ c_4=-38720\alpha-\dfrac{4725}{1156}, & c_5=8736\alpha+\dfrac{1995}{2312}, \\ c_6=-1083\alpha-\dfrac{175}{1734}, & c_7=57\alpha+\dfrac{35}{6936}. \end{cases}$$

$$\begin{cases} d_0=-12288\alpha, & d_1=22528\alpha, \\ d_2=-17664\alpha, & d_3=7680\alpha, \\ d_4=-2000\alpha, & d_5=312\alpha, \\ d_6=-27\alpha & d_7=\alpha. \end{cases}$$

As an example, Fig. 1.a shows the time characteristics of the ideal interpolation kernel, r_{sinc}, and the SC kernel, r_{SC}, for kernel parameters α_T = -71/83232 [11]. The spectral characteristic, H_{SC}, of the SC kernel is different from the spectral characteristic, H_{sinc} (box function), of the ideal interpolation kernel r_{sinc} (Fig. 1.b).



a) b)

Fig. 1. Characteristics of the ideal *sinc* and SC kernels (α = -71/83232): a) time characteristics (r_{sinc}, r_{SC}) and b) spectral characteristics (H_{sinc}, H_{SC}).

III. Algorithm for Optimal Kernel Parameters Estimating

The following Algorithm performs convolution interpolation of the Test signal, determines the interpolation error MSE depending on the parameters α. Optimal kernel parameter, α_{opt}, was determined by minimizing MSE(α). Algorithm is realized in the following steps:

Input: X - Test signal, N - Test signal length, M - interpolation frame length, L - interpolation kernel length, $\alpha_{\min}$, $\Delta\alpha$, $\alpha_{\max}$ - kernel parameter boundaries and iteration steps.

Output: α_{opt} - optimal kernel parameter, $\text{MSE}_{\min}$

FOR $\alpha = \alpha_{\min} : \Delta\alpha : \alpha_{\max}$,

Step 1: Construction of the SC kernel $r = r(\alpha)$ (Eq. 1)

FOR $I = N - M + 1$,

Step 2: Selecting the I-th frame:

$X_I = X(1 : I + M - 1)$,

Step 3: Estimating of $\hat{x}_I$ by applying Parametric Convolution Interpolation $\hat{x}_I = X[1:2:M] \otimes r$, where the symbol $\otimes$ stands for convolution.

Step 4: Interpolation error:

$$e(I) = X_I(L) - \hat{x}_I , \quad (2)$$

END *I*

Step 5: Mean square error of interpolation:

$$MSE_\alpha = \frac{1}{N-M+1} \sum_{k=1}^{N-M+1} \left| e(k) \right|^2 , \quad (3)$$

END α

Step 6: Minimum of the interpolation error:

$$MSE_{\min} = \min(MSE) , \quad (4)$$

Step 7: Optimal kernel parameter α_{opt}:

$$\alpha_{opt} = \arg\min_{\alpha}(MSE) . \quad (5)$$

As an example of the application of the described Algorithm, in Figs. 2. shows the interpolation error MSE(α). The optimal value of the kernel parameter, α_{opt}, corresponds to the minimum of the MSE.

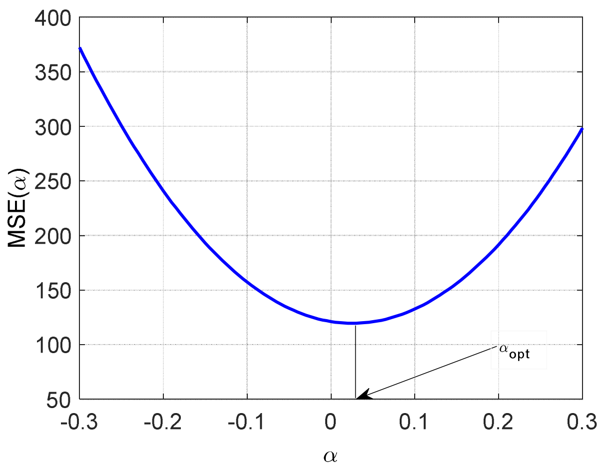


Fig. 2. The value of the optimal kernel parameter, α_{opt}, corresponds to the MSE_{min}.

IV. Experimental Results and Analysis

For the purpose of determine the interpolation error, MSE, as well as the deviation of the theoretical kernel parameter, α_T, with the kernel parameter obtained in the process of interpolation of the audio signal, α_E, an Experiment was performed.

A. Experiment

The experiment, in which the interpolation error, MSE, was determined with process of parametric convolutional interpolation, with: a) QC kernel, and b) SC kernel, was realized. The interpolation error and the optimal kernel parameter were determined using the Algorithm described in Section III. Using parametric convolutional interpolation, the audio Test signals, G1 - G7, from the Test base, were interpolated. In the second part of the Experiment, the statistical parameters (μ, σ^2) for the optimal value of the kernel parameters were determined.

Deviations of the values of the experimental kernel parameter, α_E, in relation to the theoretical values of the kernel parameters, α_T, were defined using Accuracy:

$$ACC = \alpha_T - \frac{1}{K} \sum_{k=1}^{K} \alpha_{opt}(k) = \alpha_T - \mu_\alpha , \quad (6)$$

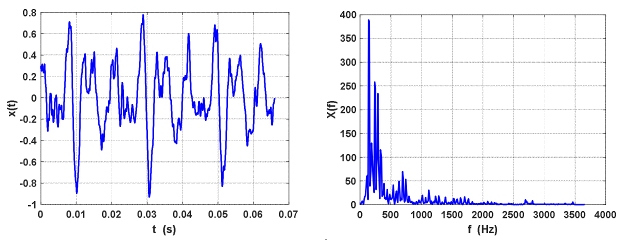
and Precision:

$$PREC = \frac{1}{K-1} \sum_{k=1}^{K} \left| \alpha_{opt}(k) - \mu_\alpha \right|^2 = \sigma_\alpha^2 , \quad (7)$$

where K is the number of the audio Test signals that are interpolated. Experimental results are presented in tables and by graphs. Finally, a comparative analysis of the results when using the QE kernel and SC kernel was performed. Theoretical optimal values for QC kernel (α_T = 3/64) and SC kernel (α_T = -71/83232) were determined in reference [11].

B. Test Base

The Test base is created from audio signals. Audio test signals were acquired by recording G tones (G1 - G7) on a Steinway B concert piano. The recording was performed in the acoustics laboratory of Iowa University. The test signals were archived on the hard disc in the form of wav files. The recording was carried out by using f_s = 44.1 kHz and 16 bps. As an example, on Fig. 3 shows the Test signal in the time and spectral domain of the tones: a) G1 (f_0 = 48.999 Hz), b) G2 (f_0 = 97.999 Hz), c) G3 (f_0 = 196 Hz), d) G4 (f_0 = 392 Hz), e) G5 (f_0 = 7833 Hz), f) G6 (f_0 = 1568 Hz) and g) G7 (f_0 = 3136 Hz) [10].



a)

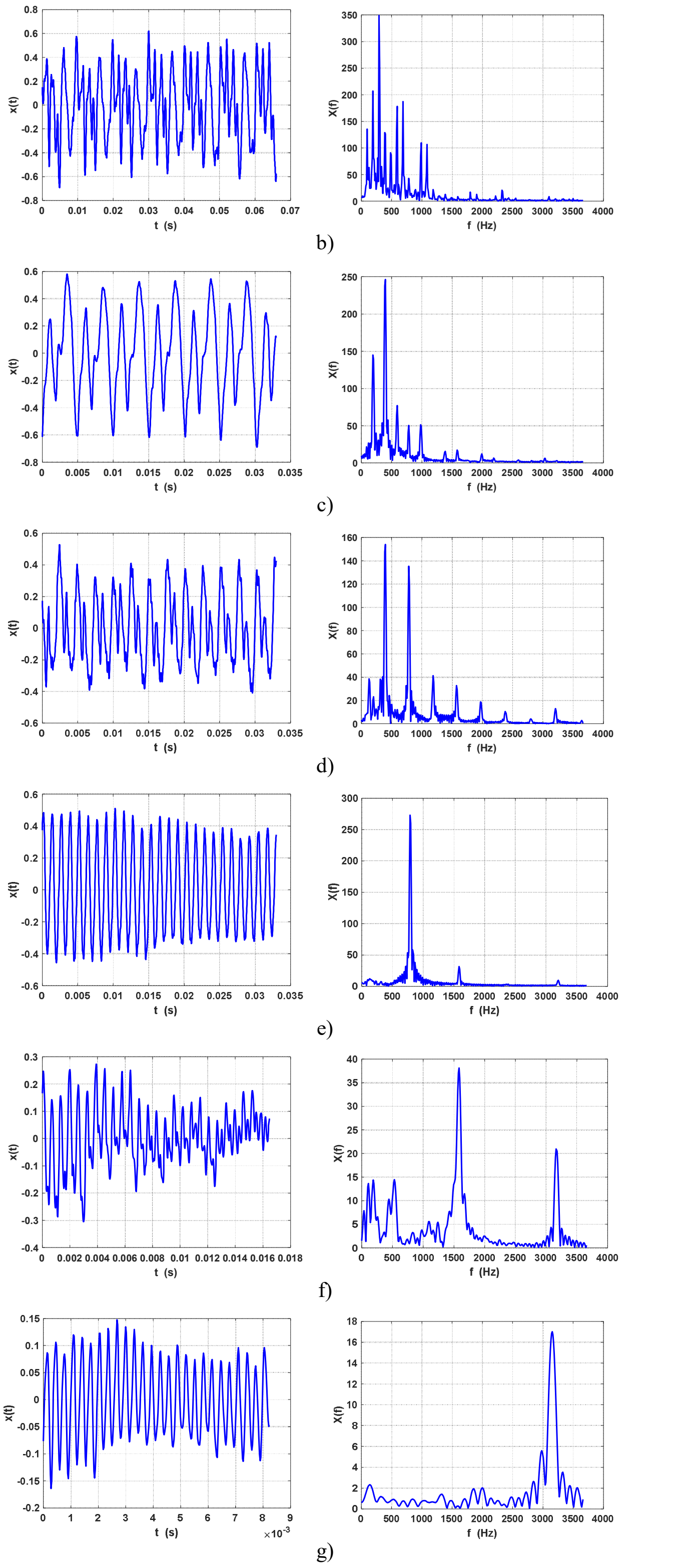


Fig. 3. Tim Audio test signal in time and spectral domain: a) G1, b) G2, c) G3, d) G4, e) G5, f) G6 and g) G7.

C. Experimental Results

Using the algorithm described in Section III, the audio Test signal interpolation was performed. Interpolation errors, MSE(α), for the audio Test signals are shown in: G1 (Fig. 4.a), G2 (Fig. 4.a), G3 (Fig. 4.a), G4 (Fig. 4.a)), G5 (Fig. 4.a), G6 (Fig. 4.a) and G7 (Fig. 4.a). The optimal values of the kernel parameters and the minimum values of the MSE(α) are shown in Table I (QC kernel) and Table II (SC kernel). In Fig. 5 shows the probability density for the kernel parameter α, when interpolated with the QC kernel (Fig. 5.a) and SC kernel (Fig. 5.a). The values for Accuracy (Eq. 6), Precision (Eq. 7) and Mean Minimum MSE (Eq. 5) are shown in Table III.

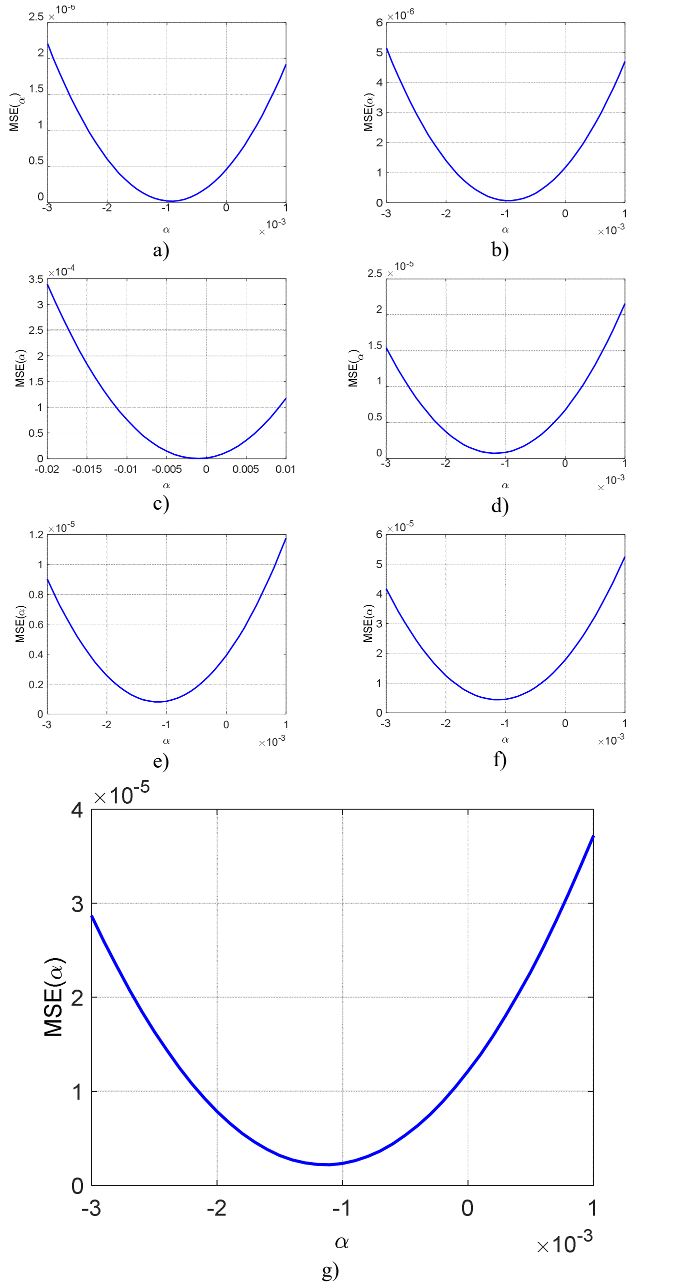


Fig. 4. MSE(α) for audio Test signals: a) G1, b) G2, c) G3, d) G4, e) G5, f) G6 and g) G7.

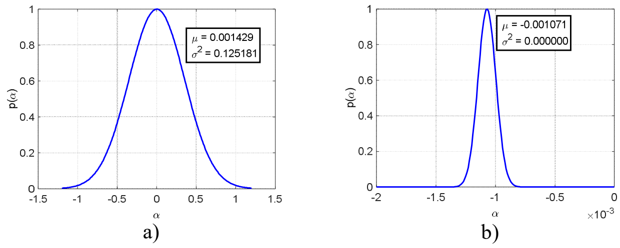


Fig. 5. Probability density: a) QC kernel, and b) SC kernel.

TABLE I. VALUES OF OPTIMAL KERNEL PARAMETERS AND INTERPOLATION ERRORS IN PARAMETRIC CONVOLUTIONAL INTERPOLATION WITH QC KERNEL

Tone	α_{opt}	*MSE*
G1	-0.2500	$8.5132\ 10^{-7}$
G2	-0.6900	$8.0338\ 10^{-8}$
G3	0.3500	$1.2904\ 10^{-6}$
G4	0.1500	$1.1485\ 10^{-6}$
G5	0.1500	$1.0576\ 10^{-6}$
G6	0.1500	$5.3607\ 10^{-6}$
G7	0.1500	$2.8782\ 10^{-6}$
		$\overline{MSE\_QC_{\min}} = 1.8096\ 10^{-6}$

TABLE II. VALUES OF OPTIMAL KERNEL PARAMETERS AND INTERPOLATION ERRORS IN PARAMETRIC CONVOLUTIONAL INTERPOLATION WITH SC KERNEL

Tone	α_{opt}	*MSE*
G1	-0.0010	$2.3442\ 10^{-8}$
G2	-0.0010	$6.8498\ 10^{-8}$
G3	-0.0010	$1.4047\ 10^{-7}$
G4	-0.0012	$7.0637\ 10^{-7}$
G5	-0.0011	$8.1499\ 10^{-7}$
G6	-0.0011	$4.4102\ 10^{-6}$
G7	-0.0011	$2.2077\ 10^{-6}$
		$\overline{MSE\_SC_{\min}} = 1.1960\ 10^{-6}$

TABLE III. ACCURACY AND PRECISION

Kernel	Accuracy ACC	*Precision PREC*	MSE_{min}
QC	0.0454	0.3538	$1.8096\ 10^{-6}$
SC	$2.4696\ 10^{-4}$	$7.5593\ 10^{-5}$	$1.1960\ 10^{-6}$

D. *Comparative Analysis*

Analysis of the experimental results, which are shown in Table 1, Table 2 and Table III, leads to the following conclusions:

a) Mean Interpolation error, $\overline{MSE\_QC_{\min}}$, when the QS kernel is used, compared to Mean interpolation error, $\overline{MSE\_SC_{\min}}$, when the SC kernel is used, is $\overline{MSE\_QC_{\min}}$ / $\overline{MSE\_SC_{\min}}$ = $1.8096\ 10^{-6}$ / $1.1960\ 10^{-6}$ = 1.513 times smaller.

b) Accuracy of the estimation of the optimal parameters, α_E, in relation to the theoretical value of the optimal parameter, α_T, when QS kernel, ACC_QC, is used, compared Accuracy, ACC_SC, when SC kernel is used, is ACC_QC / ACC_SC = 0.0454 / $2.4696\ 10^{-4}$ = 183.835 times bigger.

c) Precision of the estimation of the optimal parameters, α_E, in relation to the theoretical value of the optimal parameter, α_T, when QS kernel, is used, PREC_QC, compared Precision, PREC _SC, when SC kernel is used, is PREC _QC / PREC _SC = 0.3538 / $7.5593\ 10^{-5}$ = 4680.32 times bigger.

V. CONCLUSION

The paper presents the experimental results of the parametric convolutional interpolation of the audio Test signals. Audio Test signals are recorded G tones, which are played on Steinway B concert piano. The aim of the experiment was to determine the interpolation error, MSE, when using interpolation polynomial kernels of the fifth (QC kernel, n = 5) and seventh (SC kernel, n = 7) order. Experimental results have shown that the interpolation error when using the SC kernel, in relation to the use of the QC kernel, is 1,513 times smaller. Comparative analysis of the theoretically determined value of the optimal interpolation kernel, α_T, and the experimentally determined optimal interpolation kernel, α_E, are different. The theoretical optimization of the value of the interpolation kernel was performed with the aim of reducing the ripple of the spectral characteristic in the environment f = 0, and thus, a greater similarity with the ideal box characteristic in the environment f = 0 was achieved. However, absolute similarity of spectral characteristics with the box function has not been achieved. Therefore, there was a difference between theoretically and experimentally determined values of the kernel parameters. The efficiency of the theoretically determined optimal kernel parameters using Accuracy and Precision as parameters was analyzed. In the interpolation realized with the SC kernel, in relation to the QC kernel, Accuracy is 198,782 times bigger. When interpolating with SC kernel, compared to QC kernel, Precision is 4680.32 times bigger. The analyzed theoretical and experimental results indicate a higher efficiency of the SC kernel compared to the QC kernel, and, thus, give a recommendation for the implementation of the SC interpolation kernel in interpolation algorithms.

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