Synthesis of PID Algorithm for Speed Control of the DC Motor

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Abstract—**In this paper is proposed to synthesize the PID control algorithm for speed control of the DC motor based on the maximum stability degree method with iterations. According to the maximum stability degree method the tuning parameters depend on the maximum stability degree, which is varied and that permits to obtain high performance of the automatic control system. It was done the analytical and experimental identification of the mathematical model of the DC motor, viewed as control object. For efficacy analysis of the proposed control algorithm, there are presented case studies for control the speed of the DC motor based on the parametrical optimization method and internal control method.**

Keywords—PID controller; automatic control system; maximum stability degree method with iterations; DC motor

I. INTRODUCTION

Production processes can be classified into technological processes and information processes. Through the technological process is meant the succession of operations performed on the raw materials, semi-finished products, in order to obtain the products with the high degree of performance and quality. The operations can be divided into two main categories: the operating actions and control actions [1-2, 4].

The operating actions include activities that are necessary for evaluation of the process in concordance with the laws of physics and chemistry, which determine its development and are realized on the technological installations that involves mass and energy transfer. An important benefit for technological installations is using of electric motors, which represents the actuators elements.

Electric motors can be classified as: DC motors and AC motors. DC motors have the high starting torques, which allows them to exceed heavy loads at start-up. Another important feature of the DC motor is that the power and torque are concentrated in a compact size compared to other types of electric motors [4]. Due to the linearity of its characteristics and the relatively simple methods of speed control, DC motors are the most widely used motors as actuators elements and fall into the category of fast processes, characterized by the low time constants [8-10].

In order to fulfill correctly the operating actions and to ensure a certain quality of the production, there is necessary to be used control actions. The set of control activities forms the control process, which is performed by the control algorithm. The most useful control algorithm in different industrial applications is PID control algorithm, that becomes the standard of industrial feedback control for more than 75 years, due to its simplicity and performance that it ensures to the automatic control systems [3]. The practice of the automation the technological processes demonstrated that the PID control algorithm remains poorly synthesis in many industrial applications.

The DC motor is frequently used in the automatic control systems as an actuator element, or as a control object, that needs to control, where in both situations the problem of speed control is imposed, which is solved by the synthesis of the control algorithm [14-16].

In this paper, it was proposed to be synthesized the PID control algorithm for DC motor. In order to control the speed of the DC motor, it was proposed to be performed the synthesis of the PID controller based on the maximum stability degree (MSD) method with iterations. In order to tune the PID controller the mathematical model of the DC motor was analytical and experimentally identified. The obtained results were compared with parametrical optimization method from Matlab and internal control method [12].

II. IDENTIFICATION OF THE MATHEMATICAL MODEL

An important step in the design of the automatic control systems is the simulation and modeling of the system. The mathematical modeling of the system/control object supposes to determine the set of relations between physical variables in form of specific mathematical structures as differential equations, or transfer functions. By determining the mathematical model of a physical process, the aim is to obtain a quantitative characterization of the operation of the process as close as possible to the dynamics of the real process [1, 7, 10].

The advantages of modeling and simulation of the system/control object offer the possibility of setting regimes, that are difficult to achieve on the real installation and in providing the important information for the synthesis the control algorithm. The mathematical model of the control object can be obtained based on procedure of identification by the analytical way, or experimental way [1].

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Next, it is presented the mathematical model of the DC motor obtained by the analytical and experimental procedure of identification.

A. Analytical Identification of the DC Motor

The schematic representation of the electrical circuit of the DC motor is presented in the Fig. 1 [6].

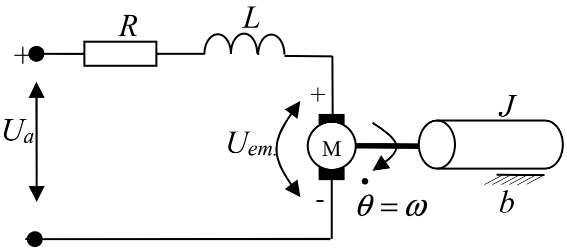

Fig. 1. Permanent Magnet DC Motor.

It was proposed to use the 2342L012 series Coreless encoder motor. The system is connected to the computer via the UART-USB module, the Atmega8 microcontroller, which receives data from the generator and transmits data to the computer.

The DC motor has the following technical parameters:

-J rotor inertia, equal with 5.7gcm^2;

-k_e back EMF constant, equal with 1.4 mV/min$^-$;

-k_m torque constant, equal with 13.4 mNm/A;

-R terminal resistance, equal with 1.9 Ω;

-k_n speed constant, equal with 713 min^{-1}/V;

-L rotor inductance, equal with 65 μH.

The input and output values of the DC motor are (Fig. 2):

- input value U_a: supply voltage;
- output value $\dot{\theta} = \omega$: rotor shaft speed;
- output value θ: rotor shaft position.

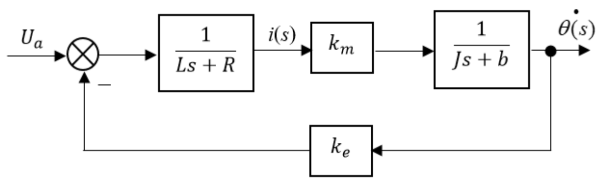

Fig. 2. The block diagram of armature controlled DC Motor.

Relationships between motor electromagnetic torque M_e and armature current i, can be expressed by [6-7]:

$$M_e = k_m \cdot i(t).$$

The induced voltage U_{em} and rotational speed $\dot{\theta} = \omega$ are:

$$U_{em} = k_e \dot{\theta}.$$

The operation of the motor is described by the equations, which were obtained by applying the second law of Newton's dynamics and, respectively, by applying the second law of Kirchhoff.

$$\begin{aligned} & J\ddot{\theta} + b\dot{\theta} = k_m i; \\ & L\frac{di}{dt} + Ri = U_a - k_e\dot{\theta}. \end{aligned} \quad (1)$$

Applying the Laplace transform to the above equations (1), it can be written:

$$\begin{aligned} & s(Js+b)\theta(s) = k_m i(s); \\ & (Ls+R)i(s) = U_a - k_e\theta(s). \end{aligned} \quad (2)$$

By removing the $i(s)$, it is obtained the transfer function in the open loop, in which the control value is the rotational speed of the rotor shaft (output value) $\dot{\theta} = \omega$ and the voltage applied to the armature windings U_a is the input value:

$$\frac{\dot{\theta}(s)}{U_a(s)} = \frac{k_m}{(Ls+R)(Js+b)+k_e k_m}. \quad (3)$$

According to the technical parameters and transfer function (3), the transfer function of DC motor can be written:

$$H(s) = \frac{\dot{\theta}(s)}{U_a(s)} = \frac{641.885}{0.00000177s^2 + 0.052s + 1}. \quad (4)$$

B. Experimental Identification of the DC Motor

In order to identify the mathematical model of the DC motor, the experimental curve of DC motor speed was raised at 5800 rpm (corresponds to the voltage of 8.96 V), Fig. 3.

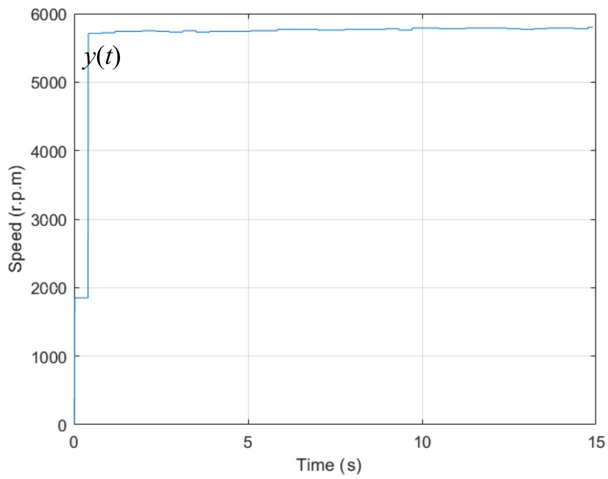

Fig. 3. The experimental curve of variation the speed of DC motor.

In the Fig. 4 is presented the static characteristic of the 2342L012 DC motor.

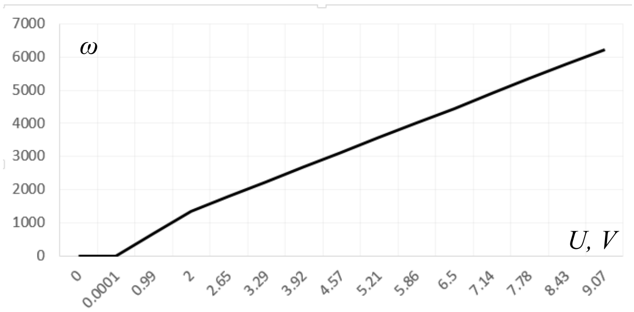

Fig. 4. The static characteristic of the DC motor.

The experimental curve presented in the Fig. 3 it was proposed to be approximated with model of object with inertia second order:

$$H(s)=\frac{k}{(T_1 s+1)(T_2 s+1)}=\frac{k}{a_0 s^2+a_1 s+a_2}, \quad (5)$$

where T_1, T_2 - time constants, k - transfer coefficient and $a_0=T_1T_2$, $a_1=T_1+T_2$, $a_2=1$.

The value of the transfer coefficient, it was calculated from the static characteristic, Fig.4:

$$k=\frac{\Delta\omega}{\Delta u}=647.32 \cdot \quad (6)$$

Next, according to the identification method presented in [13], it is calculated the value of the output speed (Fig. 3) at the level 0,632·*referencespeed*, according to this value it is calculated the time constant value T:

T=0.004 s.

According to the relationships presented in [17], there are calculated the coefficients a_0, a_1, a_2:

$a_0=0.2048T^2=0.00032$, $a_1=0.96T=0.0384$, $a_2=1$.

Based on this calculation the transfer function can be presented in the following way:

$$H(s)=\frac{k}{a_0 s^2+a_1 s+a_2}=\frac{647.32}{0.00032 s^2+0.0384 s+1}. \quad (7)$$

At the same time, it was proposed to be identified the transfer function of the DC motor using the System Identification Toolbox and it was obtained the following transfer function:

$$H(s)=\frac{k}{(T_1 s+1)(T_2 s+1)}=\frac{644.43}{(0.0209 s+1)(0.021 s+1)}= \frac{644.43}{0.00043 s^2+0.0419 s+1}. \quad (8)$$

In the Fig. 5 it is presented the comparison of the experimental characteristic, curve -1 with transient processes obtained for the case of DC motor simulation: the system with transfer function (4) – curve 2; the system with transfer function (7) – curve 3; the system with transfer function (8) – curve 4.

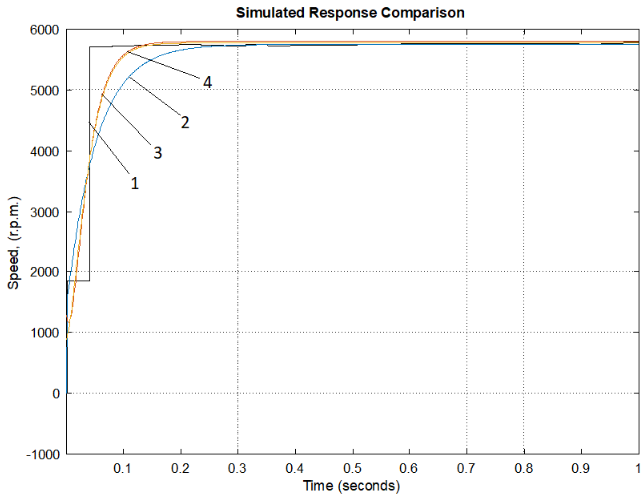

Fig. 5. Comparison of the DC motor output for different identified models.

From the Fig.5 it was observed that the best results there are obtained for the case of using the experimental identification, curves 3-4.

III. Synthesis of the PID Control Algorithm

It was proposed to tune the control algorithm PID to the identified model of object (7), which is described by the following transfer function:

$$H_{PID}(s)=k_p+\frac{k_i}{s}+k_d s, \quad (9)$$

where k_p, k_i, k_d are the tuning parameters of the PID controller [1, 3, 5].

As tuning method, it was proposed to use MSD method with iterations [11] and according to this method there are obtained the following analytical expressions for calculations the tuning parameters:

$$k_p=\frac{1}{k}(-3a_0J^2+2a_2J-a_2)+2k_dJ; \quad (10)$$

$$k_i=\frac{1}{k}(a_0J^3-a_1J^2+a_2J)-k_dJ^2+k_pJ; \quad (11)$$

$$k_d=\frac{1}{k}(3a_0J-a_1), \quad (12)$$

where J is the stability degree of the control system, Fig. 6.

According to the obtained expressions (10)-(12), there were constructed dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$, Fig. 6.

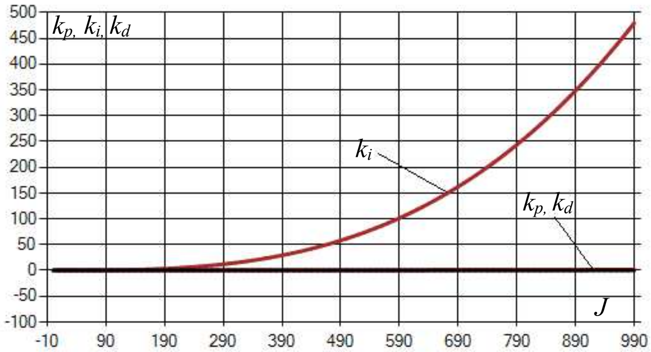

Fig. 6. Dependencies k_p=$f(J)$, k_i=$f(J)$, k_d=$f(J)$.

From the dependencies presented in the Fig. 6, there are chosen the values of the tuning parameters of the PID controller $J_i - k_{pi}$, k_{ii}, k_{di}, for different values of the stability degree, that are presented in the Table I. Based on these values, it was done the computer simulation and obtained transient processes are presented in the Fig. 7. It was done the comparison with parametrical optimization method from MATLAB and internal control method. The numbering of the curves from Fig. 7 corresponds with the numbering of the methods from the Table I.

TABLE I. THE VALUES OF THE PID CONTROLLER'S PARAMETERS

No. curves	Synthesis method	J	k_p	k_i	k_i
1	**MSD**	90	0.0021	0.061	0.0000148
2	**MSD**	370	0.003	0.106	0.00002966
3	**MSD**	590	0.01	0.36	0.000074
4	**Parametrical optimization**		0.0076	0.294	0.0000438
5	**Internal control method**		0.121	0.013	0.000708

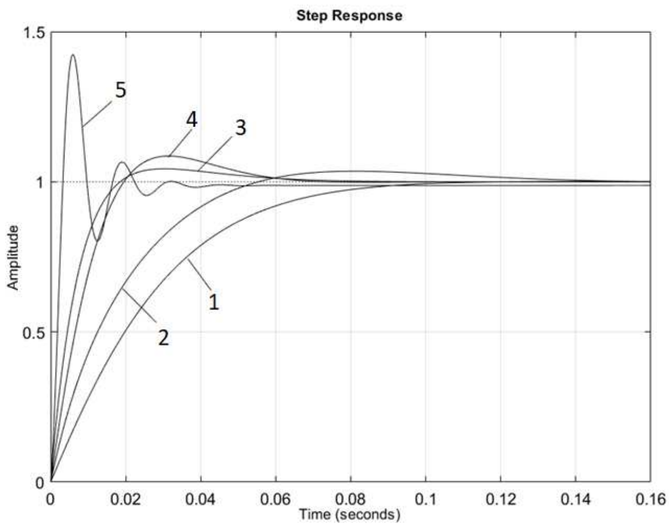

Fig.7. The transient processes obtained for the case of tuning the PID controller to the model of object (7).

In the Table II are presented the obtained performance (t_r – rise time, t_s – settling time, σ- overshoot) of the control system with different tuning parameters of the PID controller. From Table II, it can be observed that the automatic control system with PID controller tuned by the MSD method with iterations has the highest performance.

TABLE II. PERFORMANCE OF THE CONTROL SYSTEM

No.	Performance of the control system			Stability degree
	t_r, s	t_s, s	σ, %	η
1	0.0523	0.0843	0.11	40.31
2	0.036	0.111	3.5143	31.43
3	0.011	0.0511	4.31	64.79
4	0.014	0.0557	8.56	64.75
5	0.0024	0.0287	42.42	0.11

In the Fig. 8, it is presented the distribution of poles and zeros of the closed loop system in the complex plan for the case of tuning the PID controller by the MSD method with iterations, parametrical optimization and internal control method, where the numbering correspond with numbering from the Table I. According to this distribution in case of tuning the PID controller there are determinate the value of the stability degree of the system, that is indicated in the Table II for each case of tuning.

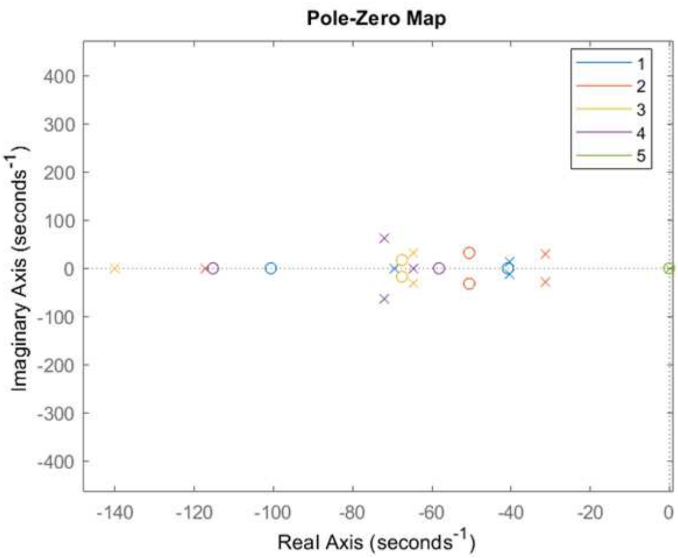

Fig.8 Distribution of poles-zeros of the closed loop system.

From Fig. 8 it can be observed that control system with PID controller tuned by the internal control method has the lowest stability degree. The MSD method by the variation the value of the stability degree - J permitted to obtain a set of tuning parameters of the PID controller, which ensure the high stability degree of the system.

IV. CONCLUSIONS

The problem of control in industrial applications is one of the most important, based on that is depending the good operation of the installations, the safety of the system in exploitation and the achievement and ensuring of the high performances.

In this paper, it was proposed to obtain the mathematical model of the DC motor by the analytical and experimental procedure of identification. The obtained results of identification were compared with experimental data. The best results were obtained in case of identification the mathematical model by the experimental way.

Based on the obtained mathematical model it was done the synthesis of the PID control algorithm according to the MSD method with iterations, parametrical optimization method and internal control method.

The designed algorithms were verified by the computer simulation and it was observed that the MSD method with iterations permitted to obtain so high performance of the automatic control system with high stability degree.

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