THE CONSTRUCTION OF THE REGULATORS FOR THE LINEAR CONROL SYSTEMS WITH THE VARIABLE PHASE SPACE MEASURABILITY

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Abstract. *Synthesis optimal in a sense of minimax is given for a regulator of linear control systems with the variable phase space measurability and with external disturbances.*
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1. Introduction

In this paper for systems with the variable phase space measurability [1, 2] and with external disturbances we propose the algorithm for synthesis optimal in a sense of minimax of the regulators. The algorithm essentially using Pontriagin maximum principle.

2. The mathematical model of the linear control systems with the variable phase space measurability

On the segment $[T_0, T_1]$ with the splitting $\{\tau_j,\ j=\overline{1,N}\}$, where $\tau_j = \{t : t \in [t_{j-1}, t_j)\}$, $j = 1,2,...,N-1$, $\tau_N = \{t : t \in [t_{N-1}, t_N]\}$, $t_0 = T_0 < t_1 < ... < t_{N-1} < t_N = T_1$, [1] let we consider the control system with the mathematic model

$$\frac{dx^{(j)}(t)}{dt} = A_j(t)x^{(j)}(t) + B_j(t)u^{(j)}(t) + G_j(t)f^{(j)}(t),\ t \in \tau_j, \quad (1)$$

$$x^{(j)}(t_{j-1}) = C_j x^{(j-1)}(t_{j-1}-) + D_j v^{(j)} \quad (2)$$

where $x^{(j)}(t)$ – n_j - is the measuring vector of the phase condition of the system (1), $u^{(j)}(t)$ – m_j - is a measuring vector of control, $f^{(j)}(t)$ – q_j - is a measuring vector of the external disturbances. $A_j(t), B_j(t), G_j(t), C_j, D_j$ – are the known matrixes with the sizes $n_j \times n_j$, $n_j \times m_j$, $n_j \times q_j$, $n_j \times n_{j-1}$, $n_j \times r_j$, according, $v^{(j)}$ – r_j - are the measuring vectors of the control parameters in relations (2), $t \in \tau_j$, $j = \overline{1,N}$.

The conditions (2) – are the ones of the structures strapping, which set the variable of the phase space measurability of the system (1) when $t = t_j$, $j = \overline{1,N}$. In relations (2) we'll suppose when $j = 1$, that $C_1 = E_1$ – single matrix of the size $n_1 \times n_1$, D_1 – zero matrix , and $x^{(0)}(t_0) = x_0^{(1)}$ – the initial condition of the system (1) when $t = t_0$.

Let the initial conditions $x_0^{(1)}$, the vectors of the external agitation $f^{(j)}(t)$, the vectors of the parameters of control $v^{(j)}$ in the structures' strapping belong to area

$$\left(x_0^{(1)}, f^{(1)}(t),..., f^{(N)}(t), v^{(1)},..., v^{(N)}\right) \in S_\lambda \quad (3)$$

where

$$S_\lambda = \Big\{x_0^{(1)}, f^{(1)}(t),..., f^{(N)}(t), v^{(1)},..., v^{(N)} : x_0^{(1)T} Q_0 x_0^{(1)} + \sum_{j=1}^{N} v^{(j)T} Q_{2j} v^j + \sum_{j=1}^{N} \int_{t_{j-1}}^{t_j} f^{(j)T}(s) Q_{1j}(s) f^{(j)}(s) ds \Big\} \le \lambda^2$$

$Q_0, Q_{1j}(t), t \in \tau_j, Q_{2j}$ – are positive defined matrixes respective sizes, $j = \overline{1,N}$, λ – some a positive constant, symbol T designates the transportation.

For the system (1) with conditions (2) let we consider the quality functional

$$I(P_1(\cdot), P_2(\cdot), ..., P_N(\cdot)) = \sum_{j=1}^{N} \int_{t_{j-1}}^{t_j} \left(\sum_{i=1}^{m_j} \sup_{S_\lambda} u_i^{(j)2}(s) + \right.$$
$$\left. + \sum_{i=1}^{M_j} \sup_{S_\lambda} \left(l_i^{(j)T} x^{(j)}(s) \right)^2 \right) ds \qquad (4)$$

with the structural choice of the control rule

$$u^{(j)}(t) = P_j(t) x^{(j)}(t), t \in \tau_j, \ j = \overline{1,N} \qquad (5)$$

In relations (4) $l_i^{(j)}$ - are the set n_j-measuring stable vectors, $i = \overline{1, n_j}$, $j = \overline{1,N}$.

Using (5), system (1) we write down as follows:

$$\frac{dx^{(j)}(t)}{dt} = \left(A_j(t) + B_j(t) P_j(t)\right) x^{(j)}(t) + G_j(t) f^{(j)}(t) \qquad (6)$$

Let $X_j(t,s)$ - are the matrix decisions of the following tasks:

$$\frac{dX_j(t,s)}{dt} = \left(A_j(t) + B_j(t) P_j(t)\right) X_j(t,s)$$
$$X_j(s,s) = E_j \qquad (7)$$

where E_j - are the single matrixes with the size $n_j \times n_j$, $s \le t$, $s \in \tau_j, t \in \tau_j$, $j = \overline{1,N}$.

Let we enter the designation:

$$W_{jk}(s,s) = X_j(t, t_{j-1}) C_j X_{j-1}(t_{j-1}, t_{j-2}) ...$$
$$... X_{k+1}(t_{k+1}, t_k) C_{k+1} X_k(t_k, s) \qquad (8)$$

$$V_{jk}(t) = W_{jk}(t, t_{k-1}) D_k$$
$$s \in \tau_k, t \in \tau_j, 1 \le k \le j, \ j = \overline{1,N} \qquad (9)$$

The decision of the system (6) with $t \in \tau_j$, which satisfies the initial requirements $x^{(1)}(t_0) = x_0^{(1)}$, can be shown as follows:

$$x^{(j)}(t) = W_{j1}(t, t_0) x_0^{(1)} + \sum_{k=1}^{j} V_{jk}(t) v^{(k)} +$$
$$+ \sum_{k=1}^{j-1} \int_{t_{k-1}}^{t_k} W_{jk}(t_k, s) f^{(k)}(s) ds + \int_{t_{j-1}}^{t} W_{jj}(t,s) f^{(j)}(s) ds \qquad (10)$$

3. Synthesis of the optimal regulator for systems of control with the variable phase space measurability

Theorem The matrixes $P_j^o(t)$, $t \in \tau_j$, $j = \overline{1,N}$, that minimize the functional (4) on the trajectories of system (6) in case of the conditions (2), determine with the formulas

$$P_j^o(t) = B_j^T(t) \Psi_j(t) \qquad (11)$$

where $\Psi_j(t)$ - is the Koshi's task solution for the matrix equation Rikkati

$$\frac{d\Psi_j(t)}{dt} = -\Psi_j(t) A_j(t) - A_j^T(t) \Psi_j(t) +$$
$$+ L_j L_j^T - \Psi_j(t) B_j(t) B_j^T(t) \Psi_j(t) \qquad (12)$$
$$\Psi_j(t_j -) = C_{j+1}^T \Psi_{j+1}(t_j) C_{j+1}$$
$$j = N-1, N-2, ..., 1, \ \Psi_N(t_N) = 0 \qquad (13)$$

The optimal value of functional (4) designates with the following formula:

$$I\left(P_1^o(\cdot), P_2^o(\cdot), ..., P_N^o(\cdot)\right) = \lambda^2 \sum_{j=1}^{N} \int_{t_{j-1}}^{t_j} \left(tr\left(L_j^T R_j(s) L_j\right) + \right.$$
$$\left. + tr\left(P_j(s) R_j(s) P_j^T(s)\right) \right) ds \qquad (14)$$

where $R_j(t)$ - is the decision of the Koshi's task for the linear matrix system with the variable structure

$$\frac{dR_j(t)}{dt} = \left(A_j(t) + B_j(t) P_j(t)\right) R_j(t) +$$

$$+R_j(t)\left(A_j^T(t)+P_j^T(t)B_j^T(t)\right)+$$
$$+G_j(t)Q_{1j}^T(t)G_j^T(t),\ t\in\tau_j,\ j=\overline{1,N} \quad (15)$$
$$R_j(t_{j-1})=C_jR_{j-1}(t_{j-1})C_j^T. \quad (16)$$

Proof For any n_j measuring vector $l_i^{(j)}$ when $t\in\tau_j$ realizes the following equality

$$\left|l_i^{(j)T}x^{(j)}(t)\right|^2=\left|l_i^{(j)T}X_j(t,t_{j-1})C_j\ldots X_1(t_1,t_0)x_0^{(1)}+\right.$$
$$+\sum_{k=1}^{j}l_i^{(j)T}V_{jk}(t)v(k)+$$
$$+l_i^{(j)T}\sum_{k=1}^{j-1}\int_{t_{k-1}}^{t_k}W_{jk}(t_k,s)f^{(k)}(s)ds+$$
$$\left.+l_i^{(j)T}\int_{t_{j-1}}^{t}W_{jj}(t,s)f^{(j)}(s)ds\right|^2=$$
$$=l_i^{(j)T}R_j(t)l_i^{(j)}\lambda^2. \quad (17)$$

Using (17), it is easy to receive the matrix equation (15) with the conditions of the phase space variable measurability (16) for all $j=\overline{1,N}$ for the determination of $R_j(t)$ when $t\in\tau_j$.
The functional (4) taking into account (17) we'll show as follows:

$$I(P_1(\cdot),P_1(\cdot),\ldots,P_N(\cdot))=\lambda^2\sum_{j=1}^{N}\int_{t_{j-1}}^{t_j}\left(tr\left(L_j^TR_j(s)L_j\right)+\right.$$
$$\left.+tr\left(P_j(s)R_j(s)P_j^T(s)\right)\right)ds, \quad (18)$$

where $R_j(t)$ - are the matrix decisions of the task (15), (16).
Let we solve the task of the functional (18) minimum finding in case of conditions (15), (16).
Let we shown the function of Gamilton-Pontriagin

$$H(R_1,\ldots,R_N,P_1,\ldots,P_N,\Psi_1,\ldots,\Psi_N,t)=$$
$$=-\sum_{j=1}^{N}\left(tr\left(L_j^TR_j(t)L_j\right)+tr\left(P_j(t)R_j(t)P_j^T(t)\right)\right)+$$
$$+\sum_{j=1}^{N}\left(tr\left(\Psi_j^T(t)\left(\left(A_j(t)+B_j(t)P_j(t)\right)\right)R_j(t)+\right.\right.$$
$$\left.\left.+R_j(t)\left(A_j^T(t)+P_j^T(t)B_j^T(t)\right)+G_j^T(t)Q_{1j}^{-1}(t)G_j(t)\right)\right),$$

where the matrix functions $\Psi_j(t)$ are the decisions of the system (11) in case of conditions (12).
Following the principle of maximum from the equation
$grad_{P_j}H(R_1,\ldots,R_N,P_1,\ldots,P_N,\Psi_1,\ldots,\Psi_N,t)=0$, $j=\overline{1,N}$, we find that the optimum decisions of the matrixes $P_1^o(t),P_2^o(t),\ldots,P_N^o(t)$ in fact determine with the formulas (10). The theorem is proved.

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