# OPTIMIZING OBSERVATION PARAMETERS USING SYSTEM FINAL STATE

**Mykola RUSNAK**
*Yury Fedkovich National University of Chernivtsi*
*str.Universietska nr.28, UA-58012 Chernivtsi*
*mpcc@chnu.cv.ua*

***Abstract.*** *In this paper we are continue to study the problems of minimax estimation linear functional under solutions systems of equations with partial derivations. New systems and new observe operators are considerate. New results are obtained in case unknown functions belong to Hilbert spaces with special metrics.*
***Keywords:*** *minimax estimation, estimation error, functional minimization.*

## Introduction

In this paper we consider the problem of minimizing a priori error of estimation of the state of systems described by linear parabolic equations. The solution of this problem we are using for the optimal constrain the parameters of the observer.

## Mathematical model of the problem

Let $\Omega \subseteq R^n$ be an open bounded set (domain) with sufficiently regular boundary $S$. Consider the sets $Q=\Omega\times(t_0,t_1)$ , $\Sigma = S\times(t_0,t_1)$ , where $(t_0,t_1)$ is an open interval in $R^1$. Let functions $a_{ij}(t,x),\ i=\overline{1,m},\ j=\overline{1,m}$ be defined on $Q$ such that $a_{ij}(t,x)\in L_\infty(Q),\ \sum_{i,j=1}^{m} a_{ij}(t,x)\xi_i\xi_j \ge \alpha\sum_{i=1}^{m}\xi_i^2$, where $\alpha>0$ for all $\xi_i\in R^1$ almost everywhere in $Q$.

Consider the Sobolev space $W_2^1(\Omega)$ - of generalized functions. Denote by $H_+$ the closure of the space. Define a continuous bilinear form $a(t,\varphi,\phi)=\sum_{i,j=1}^{m}\int_\Omega a_{ij}(t,x)\frac{\partial\varphi}{\partial x_i}\frac{\partial\phi}{\partial x_j}dx$ on the elements $\varphi,\ \phi\in W_2^1(\Omega)$. The bilinear form is corresponding by the following continuous linear operator $A(t)\varphi=-\sum_{i,j=1}^{m}\frac{\partial}{\partial x_i}(a_{ij}(t,x)\frac{\partial\varphi}{\partial x_j})$.

Under above assumptions, in the space $L_2(t_0,t_1,H_+)$ the equation

$$\frac{\partial\varphi}{\partial t}+A(t)\varphi=f_1$$

$$\varphi(t_0,x)=f_0(x),\ \varphi|_\Sigma=\overline{f}_0(t,x) \qquad (1)$$

for arbitrary $f_1\in L_2(Q)$ , $f_0\in L_2(\Omega)$ , $\overline{f}\in L_2(\Sigma)$ has a unique solution $\varphi\in L_2(Q)$. The equation (1) describes the behavior of some dynamical system.

We observe a vector $y\in L_2(\Omega)$ where

$$y=C(m)\varphi(t_1)+f_2(x), \qquad (2)$$

$$C(m)\phi=\int_\Omega m(x,\xi)\phi(\xi)d\xi,\phi\in L_2(\Omega)$$

$$m(x,\bullet)\in L_2(\Omega), m(\bullet,\xi)\in L_2(\Omega).$$

The function $m(x,\xi)$ are controlled, they are supposed to be known for arbitrary $f_2\in L_2(\Omega)$, and they describe the way we observe the dynamical system.

We consider a problem of how to estimate the state of the dynamical system (1) and then to minimize the estimation error by suitable choice of the control function $m(x,\xi)$.

To solve the problem we need some additional (a priory) information about perturbations affecting the observer and the system under investigation.

Suppose, that the perturbations $f_0$, $\overline{f}_0$, $f_1$, $f_2$ belong to an ellipsoid in the space $L_2(\Omega)\times L_2(\Sigma)\times L_2(Q)\times L_2(\Omega)$, that is

$$\int_\Omega q_0^2(x)f_0^2(x)dx+\int_Q q_1^2(t,x)f_1^2(t,x)dxdt+ \int_S q_2^2(x)f_2^2(x)dx+\int_\Sigma \overline{q}_0^2(x)\overline{f}_0^2(x)dx\le 1 \qquad (3)$$

where $q_0(x)$ , $\bar{q}_0(x)$ , $q_1(t,x)$ and $q_2(x)$ are given additional functions on the sets $\Omega$, $\Sigma$ and the interval $(t_0,t_1)$, respectively.

Then we can solve the problem of minimax estimation of a continuous linear functional

$$l(\varphi)=\int_Q g(t,x)\varphi(t,x)dxdt+\int_\Omega g_1(x)\varphi(t_1,x)dx$$

defined on the states of dynamical system (1) with observations (2), and thereafter we can determine the estimation error (see [1]).

Consider solutions $z(t,x)$ and $p(t,x)$ of the following set of equations

$$\begin{cases} -\frac{\partial z}{\partial t}+A^*(t)z=g(t,x), \\ z(t_1,x)=g_1(x)+C^*(m)q_2^2(x)C(m)p(t_1,x), \\ \left.\frac{\partial z}{\partial n_A}\right|_\Sigma=0, \\ \frac{\partial p}{\partial t}+A(t)p=q_1^{-2}(t,x)z, \\ p(t_0,x)=q_0^{-2}(x)z(t_0,x), \\ p|_\Sigma=\bar{q}_0^{-2}z|_\Sigma. \end{cases} \qquad (4)$$

Using the solutions (4), the minimax estimation error for a continuous linear functional

$$l(\varphi)=\int_Q g(t,x)\varphi(t,x)dxdt+\int_\Omega g_1(x)p(t_1,x)dx \qquad (5)$$

defined on the states of dynamical system, can be represent as a functional

$$\sigma^2=\int_Q g(t,x)p(t,x)dxdt+\int_\Omega g_1(x)p(t_1,x)dx \qquad (6)$$

Introduce a functional

$$I(m)=\sigma^2(m) \quad (7)$$

defined on the set $V_m=\{m(x,\xi):|m|\le 1,(x,\xi)\in\Omega^2\}$ and consider the problem of minimizing the functional.

**Main results**

Now we are interested to know when the problem has solutions and what necessary conditions are satisfied by the solutions.

**Theorem 1**. The set $\arg\inf(I(m))$ is nonempty. If $\bar{m}\in\arg\inf(I(m))$ , then for an arbitrary $m\in V_m$

$$\int_\Omega p(t_1,x)(C^*(m-\bar{m})q_2^2(x)C(\bar{m})p(t_1,x))dx\ge 0 \quad (8)$$

**Proof.** To prove the existence of solutions of the problem, we show that for the functional (7) some minimizing sequence converges. The embedding $H_+\subset L_2$ is compact, and the observation operator is bounded in the metric of the corresponding space. Hence, the weak convergence of a minimizing sequence in $V_m$ implies the strong convergence of corresponding solutions of the system (4) in the space $H_+$.

To obtain the necessary conditions (8), calculate in explicit form the Gateaux derivative of the functional (7).

**Examples**

Using the Theorem 1, we can solve some practical problem. For example, suppose that the control function $m(x,\xi)$ belong to the set $V_m$ and we can to observe state of the dynamic system (1) in the moment $t=t_1$.

Then (8) can be rewritten as

$$\inf\left[\int_\Omega \bar{m}(x,\eta)p(t_1,\eta)d\eta\int_\Omega m(x,\xi)p(t_1,\xi)d\xi\right]=$$

$$=\left[\int_\Omega \bar{m}(x,\eta)p(t_1,\eta)d\eta\right]^2$$

Assuming that $p(t_1,x)\ge 0$ on $\Omega$ we obtain $m(t)=\pm 1$.

**References**

[1] Наконечный, А.Г. (1985) *Минимаксное оценивание функционалов от решений вариационных уравнений в гильбертовых пространствах.* Киев, 83с.