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A Fuzzy Approach Regarding the Optimization of Statistical Process Control through Shewhart Control Charts

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***Abstract* — The main goal of most organizations, no matter of their nature, object or size, is to be competitive as possible on the market, a crucial factor in ensuring a long operating duration. Unfortunately, the appearance of variation is a very bad and undesirable thing that determine a decrease in the companies productivity. Managing and providing a better view competitiveness can not be given unless we use some statistic models. This models' variables follow closely each step of the process. If in the past years the issue of the control managing of a process wasn't seriously taken into consideration, today more things like more pretentious customers or the growth of the competitiveness level on the products and logistics market, made almost all companies to hire people, especially for the control of the quality. They have to check not only the final products but also the intermediate stages of the process. The aim of this paper is to realize a fuzzy approach based on statistical control techniques concerning on Shewhart control charts. The relationship between statistical process control and fuzzy modeling is a very researched field because the sensitivity of the fuzzy systems is an important advantage in quality analysis of the production process.**

***Index Terms* — statistical process control, control charts, special causes, fuzzy logic, double warning limit**

I. INTRODUCTION

Statistical process control (SPC) represents, as defined by Oakland [1], a set of tools for managing processes, and also for determining and monitoring the quality of final products within an organization. Also, the statistical process control can be viewed as a strategy for reducing variation in production processes, where the variation represents an unwanted thing for any company producing goods or providing services. SPC takes into account not only statistics or control, but competitiveness. The organizations compete on three important issues: quality, delivery and price. Quality is defined to be *the meeting of the requirements of the customer*. The ability to meet the customer requirements is very important and must to be in every department, every office, every workgroup for the companies that produces goods for the peoples [1,2,3]. Another well known specialist, D.C.Montgomerry [2], considered that statistical process control is a collection of very powerful techniques for solving the problems that are used to achieve the highest level of stability of the process and also in order to improve the process capability, by reducing the variability. Motorcu [3] considers statistic process control as a powerful method to measure, classify, analyze and interpret process data to improve the quality of products and service by detecting instabilities and justifying possible causes. SPC tools provide a graphical display of quality characteristics and data series versus the sample number or time. Shewhart control charts monitors if a process is or not in statistic control [4]. The variation can occur anytime and anywhere: in production, in delivery process, in people's attitude, in equipment and in it's use and in maintenance practices. The Total Quality Management (TQM), as well as the statistical process control require the process to be continuously improved by reducing variability [5]. There is two types of variation. The first type of variations are *random variations*, so that the process does don't need to be revised. This type of variation represents the sum of effects of complex interactions of random or common causes. When the random variations are not accompanied by other types of variations is practically impossible to track their causes. E.g. the set of common causes that produces variations in processes quality may include random variations in processes inputs: atmospheric pressure or temperature changes, tracking traffic on the road or power fluctuations or moisture. When in a process are present only common causes, the process is considered to be "stable", "in statistic control" or simply "in control" [6]. Unfortunately, the random variations are not the only type of variations. Another class of variations are *the variations determined by special causes*. When in *the variations that are determined by the special causes* a process have special causes of variation, the variation is considered to be "in excess" and the process is classified as "unstable" or "outside the statistical control" or beyond the expected random variations. Special causes include a faulty handling or unjustified adjustment of the process, when it is stable.

There is three areas of importance in an Shewhart control charts:

- *the stable zone* (that meets the *central line*) where there are only common causes of variation, is the area when it should not take any measures, the process is in control;
- *the warning zone* (where there are *the upper warning limit* - UWL and *the lower warning limit* - LWL) where the process can show special causes of variation and the optimal solution is to examine in more detail the process and to get more information;
- *the action zone* (where there are *the upper action limit* - UAL and *the lower action limit* - LAL) where there are only the special causes of variation and the

only solution is to investigate and adjustment the process.

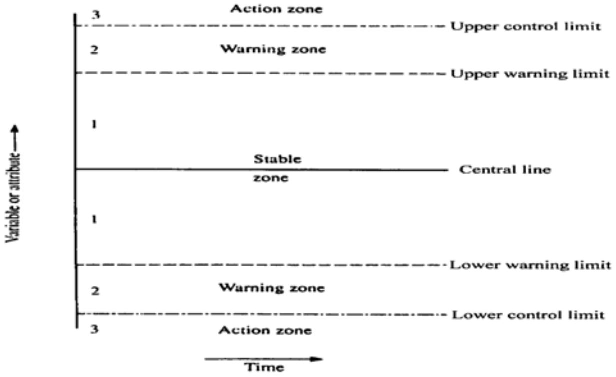

Figure 1. Schematic control chart.

In other analyses, only two areas and three lines are taken into account: the centerline that correspond to in-control state, and two other lines, upper control limit (UCL) and lower control limit (LCL). This lines replaces the UWL and LWL and also the UAL and LAL and are chosen to assure that if the process is in-control state, nearly all of the sample points will fall between them. As long as the points "fall" within the control limit, the process is said to be in-control, the condition when the outputs of the process have low variability around their target values. Shewhart charts are very powerful tools to detect process shifts.

Page[7], Roberts [8] and Western Electric [9] suggested a set of decision rules in order to detect nonrandom patterns of control charts:

Thus, with this general notions referred to statistical process control, the aim of this paper is to propose a fuzzy approach based on various rules method.

II. RELATIONSHIP BETWEEN FUZZY MODELING AND SHEWHART CONTROL CHARTS

Systems modeling is an important objective in many engineering fields and also in other areas of research [10]. Conventional approaches of systems modeling are based usually on mathematical and statistical tools that using only a precise description of each of the information used. The use of such mathematical tools (differential equations, transfer functions, very complicated integrals, Fourier transform) is well justified in the case of simple systems or well defined systems. Solid alternatives are the fuzzy modeling approaches. This relatively new approach is characterized by errors tolerance properties and enable or realize the parallel computing. The fuzzy systems are capable to emulate the imprecision and incertitude that are characteristic to human behavior. Also, this can intervene in most of real world applications, using for this a number of *if-then* rules. The fuzzy systems depend largely on the acquisition process from the human expert and have serious problems of adaptation. So, we can say that a fuzzy expert system is an expert system designed to replace a human expert in some areas of interest and is composed by three components: the base of knowledge, the inference motor and the user-interface. The knowledge of a human expert are represented by the if-then rules, the knowledge base is structured in rules and facts.

In the rows below we try to demonstrate how fuzzy logic and the SPC techniques can be in a relationship and detect quickly the situations when the process is not in-control. So, for achieving this we fuzzified and represented the run rules criteria and adaptive sampling rules. Based on the documentation of statistical process control, we developed that a run of seven consecutive points or more in a single direction is a very bad signal. However the probability of occurrence of such a run has a very low probability of occurrence in a random sample of points. Also, most of the nonrandom patterns can be detected by analyzing the last seven or eight observations from the process. In any of the control chart, each observed point leads to one or two of the next three actions: stopping the process, increase/decrease next sample size and increase/decrease next sample interval. So, based on these three actions, it can be defined three output variables.

In our application there are two types of fuzzy rules:

- rules varying the sample size and the sampling interval based on adaptive sampling concepts;
- rules identifying the process state (in or out of control) based on run rules concepts.

We have two alternatives: in the first alternative we consider the zone between central line (CL) and UWL as the *normal zone*, the zone between UWL and UAL as the *upper warning zone I* and the zone between UAL and control limit as the *upper warning zone II*. The second alternative is to increase the normal zone. So, the normal zone is considered the zone between CL and UAL. We have a single upper warning zone, i.e. the zone between UAL and control limit.

Thus, we used in the fuzzy modeling the first alternative, i.e. the statistical double warning scheme, because this is more complete and the outputs are more sensitive. The membership functions are: *lower warning zone II, lower warning zone I, normal zone, upper warning zone I* and *upper warning zone II*. Thus, a number of five fuzzy sets are used in order to represent the double warning limit sampling zones. So, this is the antecedent part of the fuzzy modeling.

From the basic theory of statistic process control [2], if we have a small sample size and a long sampling interval then we don't have shifts in the process. But a long sampling interval is synonymous with a big tolerance and this thing lead to a poor performance of the process. Also, a medium sampling interval and a large sample size provided small shifts. Finally, we can say that a medium sample size and sampling interval are to be used for large shifts. If a sample value fall completely in normal zone, then a small sample size and short sampling interval are used for the next sampling. If a sample value falls either in lower warning zone I either in upper warning zone II, then a large sample size and medium sampling interval are used for the next sampling. If a sample value falls either in lower warning zone I either in upper warning zone II, then a medium sample size and short sampling interval are used for the next sampling [11].

Western Electric [9] developed a number of four run rules based on the zones *–A*, *–B*, *–C*, *C*, *B*, *A* defined on the control chart. In the next rows we present the essence of this paper, i.e. the relationship between statistic control charts and fuzzy logic. Wang and Rowlands [12] used triangular fuzzy membership functions in order to represent the areas of interest: *OUT*, *–A*, *–B*, *–C*, *C*, *B*, *A*, *OUT*. These are the zones corresponding to Shewhart control charts. Conforming the Western Electric [9] there is a number of four rules that are presented in the next table:

TABLE I. WESTERN ELECTRIC RULES

Rule no.	**The rule description**
1	One point outside the area of UCL or LCL
2	Two out of three consecutive points in zones *–A* or *A*
3	Four out of five consecutive points in zones *–A*, *–B*, or *B*, *A*
4	Eight out of nine consecutive points in zones *–A*, *–B*, *–C* or *C*, *B*, *A*

So, it's important to say something about each of the rule. So, the rule number 1 is associated with areas of a control chart that are outside of the LCL and UCL, that are represented by the fuzzy sets – *fOUT* and *fOUT*. The rule number 2 is associated with the zones *–A* and *A* which are represented by the fuzzy sets – *fA* and *fA*. The third rule is associated with the zones *–A*, *–B*, *B*, *A* in the control chart and is represented by the union of the fuzzy sets – *fA*, *–fB* and *fA*, *fB*. The fourth and last rule is associated with the zones *–A*, *–B*, *–C*, *C*, *B*, *A* in the control charts and is represented by the union of fuzzy sets – *fA*, *–fB* – *fC* and *fC*, *fB*, *fA*. It's easy to observe that the fuzzy rule 1 is a subset of fuzzy rules 2, 3 and 4, fuzzy rule 2 is a subset of fuzzy rules 3 and 4 and fuzzy rule 3 is a subset of fuzzy rule 4. This property lead to an aggregation of fuzzy rules 1-4 to be identical to fuzzy rule 4. But, these rules should not be aggregated because the fuzzy rule base cannot be compressed.

So far, we presented the antecedent part. In the next rows we present the consequent part and the number of rules. The rule number 1 has only one point that is outside UCL or LCL. Two fuzzy rules are needed because there is one permutation for each side of the control chart. The rule number two has three points and the location for the last point must be in the zone *A* or *–A*. Two other points remain and one of them must be in *A* or *–A*. Thus, there are two permutations for each side and a total of four fuzzy rules are needed. The third rule has five points and the location for the last point must be in *A*, *B* or *–B*, *–A*. Four other points remain and three of them should be in *A*, *B* or *–B*, *–A*. Thus, there are four permutations for each side and a total of eight fuzzy rules are needed. The fourth rule has eight points and all of them must be on one side of the center line. There is only one permutation for each side and a total of two fuzzy rules are needed. So, these four rules have a total number of 16 permutations and 16 fuzzy rules are needed.

In statistic process control methodology the in-control state is represented by the binary variable 0 and the out-of-control state is represented by the binary value 1. The quality of the process or the product is consider as a fuzzy variable, named "Out", which can take any value from the interval [0,1]. As we stated above, a value near 0 show that the process is almost in-control and a value near 1 show that the process is almost out-of-control.

III. THE FUZZY RULE BASE

In this work we take into account two types of rules: the rules which specify *the next sample size* and also *the next sampling interval* and the rules which show *the state of the process*. C.Lee [13] proposed the last eight observations to be taken into account as the fuzzy inputs. So, we compare the last observation, X_8 with the fuzzy run rule number 1. The observations X_6, X_7 and X_8 will be compared with the fuzzy run rule number 2. The observations number X_4, X_5 X_6, X_7 and X_8 will be compared with the run rule number 3 and all of the observations (X_1, …,X_8) will be compared with the rule number 4. So, as we written above we have a number of eight inputs, five triangular membership functions: *lower warning zone II*, *lower warning zone I*, *normal zone*, *upper warning zone I* and *upper warning zone II*. The fuzzy sets for the output *NSZ* (*next sampling size*) are: *SSS* (*short sampling size*), *MSS* (*medium sampling size*) and *LSS* (*large sampling size*). The fuzzy sets for the output *NSI* (*next sampling interval*) are: *SSI* (*short sampling interval*), *MSI* (*medium sampling interval*) and *LSI* (*large sampling interval*). Finally, the membership functions for the output *PS* (*process state*) are: *IN* (in control process) and *OUT* (out of control process).

In the next figure we show some of the fuzzy rules based on the input variable X_8:

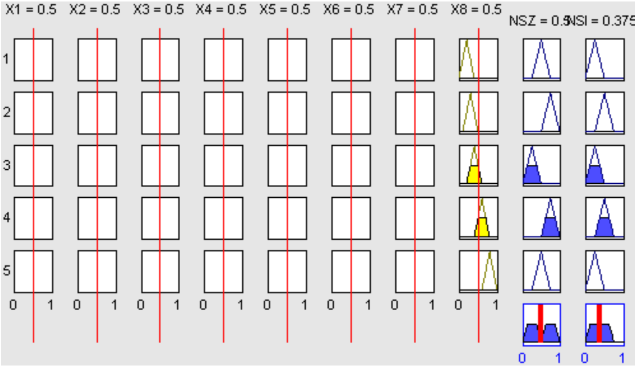

Figure 2. The rules base in the case of the X_8 observation.

So, in this rules base we don't consider nothing about the third output of the system, the process state (PS)

1. If (X_8 is in LWZII) then (NSZ is MSS) and (NSI is SSI)
2. If (X_8 is in LWZI) then (NSZ is LSS) and (NSI is MSI)
3. If (X_8 is in NZ) then (NSZ is SSS) and (NSI is SSI)
4. If (X_8 is in UWZI) then (NSZ is LSS) and (NSI is MSI)
5. If (X_8 is in UWZII) then (NSZ is MSS) and (NSI is SSI)

Now, we presented some of the rules from the rules base dedicated to the process state.

1. If (X_8 is in - *fC*) then (PS is OUT)
2. If (X_8 is in *fC*) then (PS is OUT)
3. If (X_7 is in -*fB*) and (X_8 is in - *fB*) then (PS is OUT)
4. If (X_6 is in -*fB*) and (X_8 is in - *fB*) then (PS is OUT)
5. If (X_7 is in *fB*) and (X_8 is in *fB*) then (PS is OUT)
6. If (X_6 is in *fB*) and (X_8 is in *fB*) then (PS is OUT)
7. If (X_5 is in -*fA*) and (X_6 is in - *fA*) and (X_7 is in -*fA*) and (X_8 is in -*fA*) then (PS is OUT)
8. If (X_4 is in -*fA*) and (X_6 is in - *fA*) and (X_7 is in -*fA*) and (X_8 is in -*fA*) then (PS is OUT)
9. If (X_1 is in -*fOUT*) and (X_2 is in - *fOUT*) and (X_3 is in -*fOUT*) and (X_4 is in -*fOUT*) and (X_5 is in -*fOUT*) and (X_6 is in -*fOUT*) and (X_7 is in -*fOUT*) and (X_8 is in -*fOUT*) then (PS is OUT)
10. If (X_1 is in *fOUT*) and (X_2 is in *fOUT*) and (X_3 is in *fOUT*) and (X_4 is in *fOUT*) and (X_5 is in *fOUT*) and (X_6 is in *fOUT*) and (X_7 is in *fOUT*) and (X_8 is in *fOUT*) then (PS is OUT).

The observations X_1 to X_8 correspond to the last eight observations from the process. The fuzzy sets for the input variables are the zones from the antecedent part which is defined by extended double warning limit adaptive sampling scheme [13]. The fuzzy sets – *fOUT, fOUT*, – *fA*, –*fB* – *fC* and *fC*, *fB*, *fA* are the zones from the antecedent part that were defined by the Western Electric [9]. The three output variables are, as we stated before, NSS, NSI and PS and these variables are defined in the consequent part. The first of two denoted the changes needed in the system parameters of the Shewhart control chart in the case of the next sample depending on the position of the last observation based on extended adaptive sampling schemes. The last output variable reflected the state of the process depending on the last eight observations from the process. Clearly, it is possible and even recommended to add more rules for the increase of the system sensitivity because for any non-random patterns is necessary a "IF-THEN" rule.

In his research work Mamdani used an inference mechanism of the proposed rule base. So, at each step of the process, will be taken into account a series of the last eight observations from the process and this observations will be introduced as the system inputs. Each rule has a specific degree of membership, then based on the calculated degree of membership each rule produces a number of three membership functions in the consequent part. The output membership functions from each rule will produced, by aggregation, a number of three membership functions, as we stated above: NSS, NSI and PS. The defuzzification method will produce a singles scalar quantity in order to change the Shewhart chart parameters for the system optimization. The *centroid method* is probably the most implemented defuzzification method and is done by the general formula:

$$y_i^* = \frac{\int \mu(y_i) y_i dy_i}{\int \mu(y_i) dy_i}, i = 1,\ldots,n \quad (1)$$

In our research i = 1,2,3 because we have three output variables, $\mu(y_i)$ represents the membership grade of the output membership function y_i and y_i^* is a scalar vector with three elements which represents the results of defuzzification. The out-of-control state is represented by the use of a control limit. So, if the values of the process state fall beyond the control limits the process is out-of-control.

IV. CONCLUSIONS

In this paper, based on some theoretical concepts regarding the statistical process control, we presented a fuzzy approach using the Shewhart charts in order to ensure a better competitiveness for a industrial process. The fuzzy approaches have taken significant amplitude in the last years because these are very close to human language. The main goal of the rules that are presented here is to reduce the variability of a process, the principal cause that determines a big number of scraps. As a future direction and based on this paper contributions, we will try to implement some of the systems independently at a large number of industrial processes.

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